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An econometric analysis of differences in the cost and production structure of the Brazilian trucking industry: A cluster analytic approach

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Stanford University, 1989

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AN ECONOMETRIC ANALYSIS OF DIFFERENCES IN THE COST AND PRODUCTION STRUCTURE OF THE BRAZILIAN TRUCKING INDUSTRY:

A CLUSTER ANALYTIC APPROACH

A DISSERTATION
SUBMITTED TO THE DEPARTMENT OF CIVIL ENGINEERING
AND THE COMMITTEE ON GRADUATE STUDIES
OF STANFORD UNIVERSITY
IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR THE DEGREE OF
DOCTOR OF PHILOSOPHY

by
Victor Pontes Bandeira de Mello
June 1989

I certify that I have read this dissertation and that in my opinion it is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

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ABSTRACT

Trucking defies characterization as an industry with homogeneous technology. Even when sectors within the trucking industry may be defined in terms of some common characteristic (e.g., type of shipment), markets distinguishable within sectors, with unique attributes and technical opportunities, weaken the assumption of homogeneous technological behavior. Data limitations preclude an analysis of the production structure of motor carriers that adequately accounts for the heterogeneity of their technology. There is no doubt, however, that such heterogeneity exists and is related to market characteristics; technologies are distinct in terms of transportation services demanded, with respect to both levels and types.

The present study attempts to narrow this gap by introducing a methodology to identify similar trucking firms on the basis of their cost share profiles, assuming that unobservable market and related firm operating attributes are implicit in the distribution of cost shares. This methodology, based on cluster analytic and classification tree techniques, is applied to the liquid bulk transport segment of the Brazilian trucking industry using data from 1981. Extending the traditional capital-labor-energy-material aggregation, the analysis is carried out with thirteen production factors in order to capture the interactions, at a less aggregate level, of the different types of capital, labor, and other inputs in the production of transportation services. The exploratory phase of the analysis identifies two major segments that differ with respect to the use of outside capacity. Within each segment, subgroups are also identified according to more subtle distinctions in the cost share profiles.

A set of cost models is specified and estimated to test for differences in the structure of cost and production for trucking firms in each of the subgroups. Detailed analysis using a translog flexible functional form for the cost function strongly supports the hypothesis of technical differences among groups. Apparent variation in economies of scale and in responsiveness to factor price changes reinforces the hypothesis that carriers are strongly influenced by demand requirements. In addition, through its flexible functional specification, the analysis demonstrates the inappropriateness of restrictions related to homotheticity and homogeneity of the structure of production.

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I would like to dedicate the thesis to my parents, Tutuca e Bandeira.

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Chapter I

INTRODUCTION

Over the last 35 years, the Brazilian motor carrier industry has evolved from a relatively minor position to a much more influential role in Brazil's economic structure. Currently, this industry accounts for about 55 to 60 percent of all freight transport and its total expenditures are estimated to be about 7 to 8 percent of the Gross Domestic Product. In spite of its economic significance, very little is known about the trucking industry's structure and organization. In addition to basic descriptive information, there is also need for analytic studies examining the industry's behavioral relationships and determinants. The role transportation services play in the link between social and economic functions in a country like Brazil substantiates the need for a better and more consistent understanding of the structure of trucking, its organization, and the nature of its costs of production.

The primary objective of this research is to contribute to such an understanding. Specifically, the study develops an analytical framework within which to assess the structure of technology of the road freight transport sector in Brazil. From a methodological viewpoint, it is consistent with most of the past empirical evaluations of the structure of production of transportation industries, insofar as it adheres to the basic premises of the neoclassical microeconomic theory and employs well known econometric modelling techniques. However, in order to compensate for limitations imposed by current econometric techniques and data availability, and to accommodate the particularities and institutional complexities of the trucking sector, often excluded in past analyses, the study employs a broader variety of advanced statistical techniques. In combining different modelling tools, the study demonstrates an alternative approach aimed at enlarging the link between the analytical model and the data environment.

This introductory chapter lays the necessary groundwork upon which the framework of the analysis is formulated. In Section 1.1, the complexity of the road freight market and its economic environment are discussed. The objective is to present a general description of the market's structure and dimensions, and of its socioeconomic role, which is not often fully understood by the people who participate in it. Some of the technological aspects of this industry that need to be incorporated into the development and construction of predictive models regarding the industry's behavior are also discussed. Section 1.2 is a selected review of previous empirical studies of motor carriers. The emphasis of this review centers on the assumptions underlying the adopted methodologies. These first two sections are crucial to the development of the study's analytical framework established in Section 1.3. Finally, the overall organization of this work is presented in Section 1.4.

1.1 THE TRUCKING INDUSTRY IN PERSPECTIVE

Freight transportation, as an intermediate service in the production process, provides inputs to production and finished goods to consumption centers. Freight transport demand arises through the process of production and distribution, and is consequence of consumption intensity. Each production sector needs transportation capacity to satisfy consumption requirements. The distinguishing characteristics of each production process, which are dependent upon technology, input attributes (e.g., lot size and density) and on consumption (e.g., type of good and value), determine the transportation service requirements. Transportation required by a food processing plant, for example, has characteristics distinctly different from those required by a mining firm: cost, time, and reliability of service are attributes valued differently in the transportation of perishable goods relative to the transportation of ore. Therefore, transportation services which are supplied to satisfy demands with specific attributes constitute independent markets, and incidentally, are not vulnerable to competition from other transportation modes.[1]

The process of producing freight transport depends upon the type of service. In the case of trucking, the most significant distinction lies between carriers of general commodities and carriers of specialized commodities. The former type is characterized by a large proportion of less-than-truckload shipments (LTL), small loads, and a high level of terminal and consolidation activities. In contrast, carriers of specialized commodities are distinguished by full truckload shipments (TL), large loads, and relatively little terminal or consolidation activity.

Transportation of general commodities requires terminal and consolidation activities, pick-up and delivery services, and an accurate operations planning of route and fleet dimensions. Managerial response is essential in this type of service. On the other hand, in truckload operations, planning depends mainly on the demand for services. Transportation of agricultural products, for example, is subject to production seasonality, while that of industrial products is subject to the economic situation dictating demand intensity.

Trucking firms are constrained to the network structure and to the particularities of the market in which they operate. Long distance hauls have equipment, labor and fleet requirements which differ from those in short-haul traffic. Shipment sizes, distance, load divisibility, and intensity and use of terminals qualify the type of operation in

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[1] In general, this is correct as a normative statement: demand attributes provide good assignment of shipments to modes. However, there is still some modal split within markets, which can be explained by the economies of scope in selecting a specific mode.

transportation. The economies of scale faced by firms operating mainly in long distance are not the same as those of firms operating in short distance.[2]

Motor carriers often contract capacity from the independent trucker. The use of contracted capacity appears to be more accentuated in the long haul rather than in the short. Firms tend to maintain a registry of independent truckers, offering them support services in periods of slack demand in order to assure their services in periods of excess demand.[3]

Markets are differentiated by commodity attributes and spatial flow patterns. Technologies are differentiated by size, relative efficiency, labor requirements, etc.. Some technology attributes serve specific market characteristics (e.g., refrigeration), but in general, a trucking company with a particular set of equipment will not be perfectly matched to an existing market. Thus, there are two separate differentiation schemes, and the need to balance them contributes greatly to the complexity of the trucking industry behavior.

In summary, the complexity of the freight market makes it difficult to gather and synthesize information about the industry and to construct predictive models about its behavior. Yet, even where *laissez-faire* ideology enjoys strong political support (as in the U.S.), many organizations desire information about the overall behavior of the trucking industry as an aid to public and private decisionmaking. Safety regulatory agencies, operators of competitive modes, financial analysts, and labor unions are typical examples of such organizations. These organizations exist virtually in all institutional contexts.

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[2] There are three types of economies of scale in transportation: economies of scope, of density, and of network configuration. Economies of scope occur when the cost of the joint production of more then one output is smaller than the total production cost for each output individually. Economies of density are present when total cost increases less than proportionally with output, *ceteris paribus*. Economies of network configuration are related to the cost savings resulting from the efficient location of terminals, concentration of traffic flows, etc., once the economies of density have been taken into account.

[3] In the United States, the relationship among freight firms and independent truckers is almost tutelary. Firms assure them a minimum profit which allows vehicle capital remuneration and a regular standard of living.

1.1.1 Market Structure

Road freight transport accounts for about 60 percent of goods movement in Brazil.[4] Although the railroad share of bulk commodities has been increasing, substantial overall change in the role of the trucking sector is unlikely because of the scarcity of capital for new rail infrastructure and because the majority of goods are non-bulk.

The Brazilian trucking industry consists of two major segments: for-hire carriers and private carriers. For-hire carriers engage in transportation (for compensation) of one or more classes of freight that is the property of others. Private carriers are individuals or firms that transport internally produced material in owned or leased vehicles. The Departamento Nacional de Estradas de Rodagem (DNER) of the Ministry of Transportation further classifies carriers into six categories:

- Commercial Freight Firms (CFF): for-hire motor carriers providing transportation capacity of more than 60 tons;
- Independent Trucker (ITS): individual truckers providing services either by direct contract with shippers or by renting capacity to a CFF;
- Private Carriers (PCF): producers transporting their own commodities and sometimes making their vehicles available for rent;
- Individual Private Carriers (IPC): owners or co-owners of one or more vehicles engaging solely in the transportation of their own commodities;
- Pick-up and Delivery Firms (PDF): carriers providing services over short distances, in vehicles up to 7 tons of net capacity;
- Truck Rental Firms (TRF): organizations renting vehicles to the CFF.

The official statistics of the industry are incomplete and inaccurate since most of the autonomous carriers are not registered and several firms are incorrectly classified. For

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[4] From Rezende [1984].

example, a PCF, which can only exist in association with a production activity, can be replaced by a CFF which acts exactly like a PCF, but will appear as a CFF in the records. The composition of the Brazilian trucking industry according to the above categories is presented in Table 1.1 for the years 1980, 1981, and 1982. The apparent rise in the number of firms and fleet size is primarily a consequence of increased registration of firms rather than sectoral growth. According to DNER, about 400,000 truckers are still not registered, mostly light trucks operating in urban areas. Nevertheless, the data are indicative of the size of each sector.

Table 1.1: Composition of the Brazilian Trucking Industry [a]

	number of firms			number of trucks			trucks/firm		
class [b]	08/80	10/81	10/82	08/80	10/81	10/82	1980	1981	1982
CFF	5087	5854	5999	78304	92471	98515	15.4	15.8	16.4
ITS	106264	146063	156791	112619	152372	159031	1.1	1.0	1.0
PCF	18758	30728	35397	148877	235062	282801	7.9	7.7	8.0
IPC	4444	13370	19041	10259	25603	34970	2.3	1.9	1.8
TRF	7086	8381	8681	19310	24204	26019	2.7	2.9	3.0
total				369369	529712	601336			

Note: a. pick-up and delivery firms (PDF) are excluded.
b. see class definition in the text.

Source: DNER, Relatório Estatístico do RTRC (08/80, 10/81, 10/82).

The total truck fleet, according to data from CNVP/SERPRO (Table 1.2), conflicts with that provided by DNER.[5] According to SERPRO, the 1983 fleet data are more

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[5] Cadastro Nacional de Veículos e Proprietários (CNVP)is a record of all licensed vehicles in the country. SERPRO - Serviço Nacional de Processamento de Dados, maintains this data base.

accurate than those of previous years due in part to the elimination of double-countings and of vehicles which had not renewed license for three consecutive periods.[6]

Table 1.2: Brazilian Truck Fleet by Fuel Type and Vehicle Size

	gasoline			diesel			alcohol	
type [a]	1981	1982	1983	1981	1982	1983	1982	1983
light	55094	59285	52391	109994	111922	121587	86	94
medium	141381	162230	119714	409972	428130	419346	1205	1317
semi-heavy	971	2933	2164	142625	142981	145866	10	35
heavy				77520	85584	81479		
super-heavy				10529	6490	7578		

Note: a. 1 < light < 10 MCT - Maximum Capacity of Traction (tons)
10 < medium < 20 MCT
20 < semi < 30 MCT
30 < heavy < 40 MCT
40 < super

Source: Cadastro Nacional de Veículos e Proprietários and Departamento Nacional de Estradas de Rodagem

Table 1.3 shows the percentages of fleet within each group of firms by service specialization. The data refer to the available fleet without regard to actual use. Assuming the data are representative of fleet specialization, there appear to be definite patterns of dominance. For example, PCFs play a large role in refrigerated shipments, live cattle, solid

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[6] It should be emphasized that the decrease in the gasoline-fueled vehicles by wreckage without replacement was a consequence of the oil price hike which led to dieselization. Government pricing policy accelerated this process in such a way that, presently, only a small percentage of commercial vehicles produced in Brazil are gasoline fueled.

bulk, and general commodities.[7] In contrast, CFFs dominate in vehicles and oil derivatives, and ITSs dominate in parcels, containers, and lumber.

Table 1.3: **Distribution of the Brazilian Truck Fleet by Type of Firm for Common Specializations[a]**

	type of firm[b]				
specialization	CFF	ITS	PCF	IPC	total
General Commodities	21.0	30.0	44.0	5.0	100.0
Solid Bulk	15.9	21.6	55.5	6.9	100.0
Parcels	21.7	48.0	27.4	2.9	100.0
Containers	21.4	50.7	25.0	2.9	100.0
Lumber	21.8	51.9	23.6	2.7	100.0
Vehicles	44.6	25.3	29.6	0.5	100.0
Cattle	22.6	20.8	45.6	11.1	100.0
Oil Derivatives	58.1	18.0	23.9	0.0	100.0
Refrigerated Shipments	34.1	5.6	59.1	1.2	100.0

Note: a. as of October, 1981.
b. see class definition in the text. Pick-up and Delivery Firms (PDF) are excluded. Truck Rental Firms (TRF) are combined with CFF.

Source: DNER, Relatório Estatístico do RTRC (10/81).

These patterns result from an interaction of technology with market and organization structure. For example, the first three PCF concentrations require highly specialized equipment (e.g., refrigerated vehicles), and entail empty backhauls, and possibly, sharp seasonality.[8] Outside carriers may be unable to compete with internally

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[7] In the transportation of general commodities the participation of CFF is apparently small. However, it is actually greater than it appears since more than 70% of independent truckers operate with general commodities via CFF.

[8] Idle return is a joint product of such specializations in transportation.

provided services which can accommodate slack periods by shifting labor assignments within the organization. In addition, some sectors, such as retailing, exhibit a tendency towards vertical integration involving transportation. This is because both origin activities (warehousing) and destination activities (retail sales) are internal to the organization; contracting for transportation between the two activities would amount to relinquishing control over a major intermediate step in the production process.[9]

Outside contracting is much more likely for inputs to and outputs from a firm's production process. Thus, a retail firm that purchases goods from a wholesaler will make use of for-hire carriage, as will a manufacturer that does not operate sales outlets. Most of the CFF and ITS concentrations can be explained by this fact. Differences between CFFs and ITSs are due in part to the spatial and temporal variability of demand. Fleets of CFF firms are often sized to handle volumes that can be anticipated with virtual certainty; independent truckers are hired to handle excess demands.

This pattern of independent contracting is a critical characteristic of the Brazilian trucking sector. ITS availability provides a device for capacity adjustment for the CFF in the short term. In larger firms with more sophisticated management, the utilization of outside capacity is intensified. For example, the São Paulo-based Expresso Araçatuba contracts services for 70 percent of its cargo. In 1980, its fleet was composed of 211 vehicles, including administration vehicles and 78 small trucks for collection and delivery. Yet, an average of over 800 different trucks made use of its freight facility each day, and more than 6,000 independent truckers were registered with the firm for intercity carriage.

The present trend in the Brazilian road freight sector is toward a CFF concentration on pick-up, delivery and consolidation activities, transferring all but the least variable long distance carriage to the owner-operator. This emerging picture reveals a trucking industry

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[9] Evidence suggests that shippers that still maintain their own transport services do so because they are not satisfied with the reliability of the professional freight firms.

dominated by CFFs as umbrella organizations, but characterized by the complex, nested decision processes of interrelated firms and operators.[10]

1.1.2 The Commercial Freight Firms

The commercial freight firms are the subject of a survey conducted annually by Fundação Instituto Brasileiro de Geografia e Estatística (IBGE). The survey covers all types of organized firms whose primary activities are to provide road transportation (passenger or freight) for hire.[11] According to IBGE, in 1981 the CFFs directly consumed 1.9 million cubic meters of diesel oil, i.e., about 10 percent of the country's total consumption.[12] Through subcontracting of owner-operators, their diesel consumption share totaled 18 percent. In that same year, the 11,000 carriers directly employed 224,000 persons and generated revenues in the order of US$ 5.3 billion.[13]

Of the 10,766 firms surveyed by IBGE in 1981 that had revenues generated from freight transportation only, as opposed to revenues from freight and passenger transport, 1857 carriers (17.25%) operated on regular lines predominantly. The remaining 83 percent provided services on non-fixed routes. Table 1.4 compares some of the main economic characteristics of these two classes of carriers.

The IBGE survey allows the classification of the CFFs according to the type of equipment used and type of operation. Equipment type is defined by cargo characteristics, i.e., whether the cargo is a dry or liquid commodity, or if it requires refrigeration. Operation type is characterized according to intracity, intercity, interstate, or international

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[10] In the State of Paraná, Centers for Freight Information (CFI) reveal that 71.1 percent of the loads commissioned to independent truckers come from the CFF. However, it is estimated that the CFF share of posted freight is more than 80 percent. The evidence of utilization of independent truckers from Expresso Araçatuba in 1980, and from the CFI in 1983, suggests the important role of autonomous capacity in the freight sector [Transporte Moderno, 1983].

[11] This survey, which is the main source of information for this work, does not include the independent trucker (other than as a cost category for organized firms). More details are given in Chapter III.

[12] In *Indicadores de Conjuntura* [Conjuntura Econômica, 6/83].

[13] Dollar values are based on an average exchange rate of Cr$/US$ 93.18 for that year, according to data published in Conjuntura Econômica of June of 1982.

freight carriages on regular lines, by services provided on non-fixed routes, or a combination of each. Table 1.5 illustrates the breakdown by type of equipment and operation of the CFFs.[14]

Table 1.4: Comparison Between CFFs With and Without Regular Lines

operation	firms	revenues $10^9 US\$$	output 10^6 *tons*	personnel	employees/ firm	revenues/ firm $10^3 US\$$	revenues/ employee $10^3 US\$$
no regular lines	8909	3.9	189	153085	17.2	433.96	25.25
regular lines	1857	1.4	44	65965	35.5	730.05	20.55
all	10766	5.3	233	219050	20.3	485.05	23.83

Source: IBGE [1984a].

There is a high degree of specialization within the sector with regard to equipment type. Almost 94 percent of the carriers are dedicated to either dry, liquid, or refrigerated shipments, as can be seen in Table 1.6. Even when more than one type of equipment is present, there is always the substantial predominance of one type over the others. The 78 percent share of firms specializing in the transportation of dry goods alone is easily understood, given the fact that dry goods require equipment supporting the largest number of specializations as well as types of cargo.

With respect to traffic lines, a high degree of specialization is also verified. About 96 percent of the sample operates one type of line haul only (Table 1.7), i.e., either carriers

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[14] A total of 398 companies did not report type of equipment and, therefore, were not included in the tables.

Table 1.5: Number of Carriers by Type of Shipment and Operation

	type of shipment [a]							
operation [b]	S	L	F	SL	SF	LF	SLF	total
5	6914	952	193	355	108	14	26	8562
40	0	1	0	0	1	0	0	2
45	2	0	0	1	1	0	0	4
300	392	108	9	15	9	0	0	533
305	38	5	0	9	1	0	1	54
340	3	0	2	0	1	0	0	6
345	1	0	1	0	0	0	0	2
2000	302	268	7	27	1	1	2	608
2005	46	6	1	17	2	0	5	77
2040	0	0	0	0	1	0	0	1
2300	142	20	1	14	4	2	2	185
2305	40	4	2	7	4	1	0	58
2340	1	1	0	0	2	0	0	4
2345	1	0	0	0	0	0	0	1
10000	85	15	3	2	0	0	0	105
10005	11	1	0	0	0	0	0	12
10300	13	2	0	1	1	0	0	17
10305	0	0	0	1	0	0	0	1
12000	50	8	2	4	2	0	0	66
12005	5	0	0	1	2	0	0	8
12300	35	6	1	5	0	0	0	47
12305	8	3	0	1	0	0	2	14
12340	1	0	0	0	0	0	0	1
no regular lines	6914	952	193	355	108	14	26	8562
regular lines	1176	448	29	105	32	4	12	1806
Total	8090	1400	222	460	140	18	38	10368

Note: a. dry (S), liquid (L), and refrigerated (F).
b. regular lines: intracity (10000), intercity (2000), interstate (300), international (40); no regular lines (5).
Source: IBGE [1984a].

Table 1.6: Distribution of Carriers According to Type of Shipment (%)

	type of shipment[a]							
operation[b]	S	L	F	SL	SF	LF	SLF	total
5	80.8	11.1	2.3	4.1	1.3	0.2	0.3	100.0
40	0.0	50.0	0.0	0.0	50.0	0.0	0.0	100.0
45	50.0	0.0	0.0	25.0	25.0	0.0	0.0	100.0
300	73.5	20.3	1.7	2.8	1.7	0.0	0.0	100.0
305	70.4	9.3	0.0	16.7	1.9	0.0	1.9	100.0
340	50.0	0.0	33.3	0.0	16.7	0.0	0.0	100.0
345	50.0	0.0	50.0	0.0	0.0	0.0	0.0	100.0
2000	49.7	44.1	1.2	4.4	0.2	0.2	0.3	100.0
2005	59.7	7.8	1.3	22.1	2.6	0.0	6.5	100.0
2040	0.0	0.0	0.0	0.0	100.0	0.0	0.0	100.0
2300	76.8	10.8	0.5	7.6	2.2	1.1	1.1	100.0
2305	69.0	6.9	3.4	12.1	6.9	1.7	0.0	100.0
2340	25.0	25.0	0.0	0.0	50.0	0.0	0.0	100.0
2345	100.0	0.0	0.0	0.0	0.0	0.0	0.0	100.0
10000	81.0	14.3	2.9	1.9	0.0	0.0	0.0	100.0
10005	91.7	8.3	0.0	0.0	0.0	0.0	0.0	100.0
10300	76.5	11.8	0.0	5.9	5.9	0.0	0.0	100.0
10305	0.0	0.0	0.0	100.0	0.0	0.0	0.0	100.0
12000	75.8	12.1	3.0	6.1	3.0	0.0	0.0	100.0
12005	62.5	0.0	0.0	12.5	25.0	0.0	0.0	100.0
12300	74.5	12.8	2.1	10.6	0.0	0.0	0.0	100.0
12305	57.1	21.4	0.0	7.1	0.0	0.0	14.3	100.0
12340	100.0	0.0	0.0	0.0	0.0	0.0	0.0	100.0
no regular lines	80.8	11.1	2.3	4.1	1.3	0.2	0.3	100.0
regular lines	65.1	24.8	1.6	5.8	1.8	0.2	0.7	100.0
Total	78.0	13.5	2.1	4.4	1.4	0.2	0.4	100.0

Note:
a. dry (S), liquid (L), and refrigerated (F).
b. regular lines: intracity (10000), intercity (2000), interstate (300), international (40); no regular lines (5).

Table 1.7: Distribution of Carriers According to Type of Operation (%)

	type of shipment [a]							
operation [b]	S	L	F	SL	SF	LF	SLF	total
5	85.5	68.0	86.9	77.2	77.1	77.8	68.4	82.6
40	0.0	0.1	0.0	0.0	0.7	0.0	0.0	0.0
45	0.0	0.0	0.0	0.2	0.7	0.0	0.0	0.0
300	4.8	7.7	4.1	3.3	6.4	0.0	0.0	5.1
305	0.5	0.4	0.0	2.0	0.7	0.0	2.6	0.5
340	0.0	0.0	0.9	0.0	0.7	0.0	0.0	0.1
345	0.0	0.0	0.5	0.0	0.0	0.0	0.0	0.0
2000	3.7	19.1	3.2	5.9	0.7	5.6	5.3	5.9
2005	0.6	0.4	0.5	3.7	1.4	0.0	13.2	0.7
2040	0.0	0.0	0.0	0.0	0.7	0.0	0.0	0.0
2300	1.8	1.4	0.5	3.0	2.9	11.1	5.3	1.8
2305	0.5	0.3	0.9	1.5	2.9	5.6	0.0	0.6
2340	0.0	0.1	0.0	0.0	1.4	0.0	0.0	0.0
2345	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
10000	1.1	1.1	1.4	0.4	0.0	0.0	0.0	1.0
10005	0.1	0.1	0.0	0.0	0.0	0.0	0.0	0.1
10300	0.2	0.1	0.0	0.2	0.7	0.0	0.0	0.2
10305	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.0
12000	0.6	0.6	0.9	0.9	1.4	0.0	0.0	0.6
12005	0.1	0.0	0.0	0.2	1.4	0.0	0.0	0.1
12300	0.4	0.4	0.5	1.1	0.0	0.0	0.0	0.5
12305	0.1	0.2	0.0	0.2	0.0	0.0	5.3	0.1
12340	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
no regular lines	85.5	68.0	86.9	77.2	77.1	77.8	68.4	82.6
regular lines	14.5	32.0	13.1	22.8	22.9	22.2	31.6	17.4
Total	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0

Note:
- a. dry (S), liquid (L), and refrigerated (F).
- b. regular lines: intracity (10000), intercity (2000), interstate (300), international (40); no regular lines (5).

do not have regular lines (type 5) or if they do, they are predominantly interstate (type 300), intercity and/or interstate (type 2000 and 2300), or intracity operations (type 10000). Moreover, there is a remarkable split between carriers characterized by fixed routes and those that are not. In 1981, for example, only three percent of the revenues of companies with regular lines were generated from services in non-fixed routes. On the other hand, those carriers without fixed routes had less than five percent of their revenues generated from services in fixed routes. Of course, a more thorough characterization of such specializations (equipment requirements and type of routes) would be even more informative. However, it is beyond that allowed by the format of the IBGE questionnaire.

In summary, the trucking industry is characterized by a multitude of segments or sectors facing distinct markets, producing different outputs, and consequently subject to distinct decisionmaking behavior, cost structure and technology. This diversity of segments makes any analysis of trucking quite complex. However, any study trying to address questions on the economic behavior of the road freight sector must consider such diversity.

1.2 COST FUNCTIONS OF TRANSPORTATION INDUSTRIES

The empirical findings from econometric studies on cost and demand related to the variety of transportation modes in general, and in particular to trucking, are characterized by systematic contradictions. According to Friedlaender and Spady [1981], these controversies stem not only from the lack of adequate costing schemes but also from the inability to specify a cost function that appropriately characterizes the technology.

In this section, some of the most recent studies on the structure of production in the motor carrier industry are examined. The purpose is not to present an exhaustive review of the literature, as this has been done in detail elsewhere.[15] Rather, this section is an attempt

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[15] See Breautigam and Baesemann [1978] for a discussion of the main findings of such studies. Also, Chow [1978] provides an extensive survey on the major studies of economies of scale in the U.S. motor freight industry.

to analyze and characterize conventional approaches to studying the economics of the trucking industry. Three points will serve as the focus of analysis: model specification, variable selection, and sample attributes.

1.2.1 Literature Review

Empirical studies of the structure of production of the motor carrier industry have invariably focused on one or more of the following aspects:

- the nature of economies of scale;
- distribution and optimal firm size;
- substitution among factors of production, and (based on these)
- effects of regulatory policies.

Although there are some studies based on production functions, analyses based on cost functions predominate: Ladenson and Stoga [1974] investigated returns to scale using a Cobb-Douglas production function; Koenker [1977], Spady and Friedlaender [1978], Chow [1978], Klem [1978], Cherry [1978], Friedlaender and Spady [1981], and Harmatuck [1981], among others, adopted different specifications of the cost function as the statistical model of analysis. Most of the reviewed works involved the regulated American trucking industry.

Included among the studies based on a production function is that by Ladenson and Stoga [1974] who estimated a Cobb-Douglas function for a cross-sectional sample of 116 general freight common carriers, in order to test the hypothesis that the scale parameter would vary according to firm size. The Cobb-Douglas specification included capital and labor as the only production factors involved in the allocation process. Dummy variables were also included in the specification to indicate firm size according to classes defined by the number of employees. The results weakly support the hypothesized conjecture that, as the firm expands, a stage should be reached where some *retarding* factors would be eliminated and a regime of constant or increasing returns would be encountered.[16]

However, there are three issues that make the validity of these results uncertain. First, the direct estimation of the production function is not an appropriate statistical model for the exogenous characteristic of production flows implied by the regulated environment.[17] Second, there is the questionable assumption that the allocation process can be characterized by only two factors — capital and labor, i.e., that the many factors in the production of transportation services can be bundled into just two aggregates. Third, there is the use of a single measure of output, ton-miles, which aggregates distinct transportation services.

Optimal scale and size distribution of trucking firms was the focus of Koenker's work [1977]. A cost function for interstate common carrier trucking firms was estimated using a time series of annual data on costs and output from a cross section of general freight trucking firms with headquarters in the central United States. The model specification was the separable form of the cost function, $C[q,p] = \alpha[q]\ c[p]$, where $\alpha[q]$ is a scaling function in the vector of output variables q, and $c[p]$ is a unit cost function in the input price vector p.[18,19] The scaling function $\alpha[q]$ qualified the aggregate measure of a firm's output by introducing variables such as average length of haul, average shipment size, and number of shipments.

Koenker exploited the homothetic production structure to simplify his estimation procedure by making one further assumption: that no price variation existed among the firms within cross-sections. Thus, the unit cost function $c[p]$ could be factored into a set of

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[16] This conjecture was first speculated by Dicer in his 1971 article. See Ladenson and Stoga [1974] for reference.

[17] It is a clear reversal in the role of endogenous and exogenous variables, with obvious implications for the model's error structure.

[18] Under the assumption that optimal factor proportions are not dependent upon their magnitude, a technology is said to be homothetic, and the cost function may be written in this multiplicative form.

[19] The original notation will be kept throughout this review.

temporal intercept terms, and the total cost function could be determined through a partial adjustment model with static expectations taking the form

$$C_{ft} = A_t + \gamma^{+} \Delta Q_{ft}^{+} + \gamma^{-} \Delta Q_{ft}^{-} + \alpha_0 Q_{ft\text{-}1} + \alpha_1 Q_{ft\text{-}1}^{2} + \beta_1 H_{ft} + \beta_2 W_{ft} + \varepsilon_{ft},$$

where, for firm f and time t, Q_{ft}, H_{ft}, and W_{ft} are the logarithms of output, average haul, and average load per trip, respectively.[20] Cost, as the dependent variable, was defined in three different ways: total costs excluding capital costs, variable costs excluding all depreciation costs for capital inputs, and direct labor costs excluding fuels, tires and administrative salaries.

The homotheticity assumed in Koenker's specification has major implications with respect to his conclusions about optimal firm size. For instance, under an input-output separable production structure, firm management does not perceive different production processes when operating in, for example, short or long haul.[21] This is very unlikely to occur.

Spady and Friedlaender [1978] introduced a hedonic cost function that can be used to take output characteristics into account, and applied it to the regulated trucking industry. Although they considered a technology of motor carriers characterized by multiple outputs, they did not use a multiproduct specification. Basically, a "hedonic cost that uses hedonic functions of outputs and qualities as their argument" was estimated, instead of using "conventional cost functions that use outputs or quality-adjusted outputs as their arguments [page 160]." However, this specification, as in Koenker's work, is restrictive since the cost minimizing factor combination is considered independent of the composition of effective output.

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[20] In order to account for the asymmetry with respect to over- and under-estimates of planned output level, the variable $\Delta Q_{ft} = Q_{ft} - Q_{ft\text{-}1}$ took two forms: $\Delta Q^{+} = \Delta Q$ if $\Delta Q > 0$, and $\Delta Q^{-} = \Delta Q$ otherwise.

[21] The implications of homotheticity are discussed in Chapter II.

The specification used is a quality-separable hedonic function given by

$$C = C[\Psi(y,q),w],$$

where $\Psi(y,q)$ is a vector of functions that measure effective outputs, and w represents a vector of factor prices (capital, labor, fuel, and purchased transportation). For the i-th physical output y^i and an r-dimensional vector of output qualities q^i, $\Psi^i(y^i,q^i)$ was approximated by a linear homogeneous translog, jointly estimated with $C = C[\Psi(y,q),w]$ also in a translog form. The technology implied by such a specification can be envisioned as the combination of input factors that produces the abstract outputs represented by Ψ^i. The hypothesis of homothetic technology was tested and rejected. Moreover, it was concluded that common carriers of general commodities are not subject to economies of scale.

The use of hedonic functions to characterize output may be a reasonable alternative in the case of a regulated industry. The nature of the U.S. Interstate Commerce Commission (ICC) regulation is such that shipment attributes and output composition may be viewed as exogenous to the carrier. Otherwise, the use of hedonic functions would lead to problems of identification, and their coefficients would be ambiguously reflecting supply or demand effects.[22]

Harmatuck [1981] discusses the major methodological problems found in estimates of motor carrier cost functions:

- the use of highly restrictive functional forms,
- the improper characterization of output,
- the omission or use of improperly measured factor prices, and
- the use of a heterogeneous sample of firms.

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[22] See Spady and Friedlaender [1978], page 162, for a more detailed discussion.

In order to counter such problems, Harmatuck suggests the specification of a cost function to a set of activities of trucking firms with the following characteristics:

- specification of a translog joint cost function with extensions that add flexibility to the functional form; [23]
- development of a multiproduct specification where LTL and TL traffics are treated as distinct products, each of which is described by the annual number of shipments, average size of shipments and average length of haul;
- rather than dealing with aggregates of labor and capital, aggregation of input factors in activity sets: line haul (vehicle-miles), pickup and delivery (tons), billing and collecting (shipments), platform handling (LTL tons), and all other factors.

The estimation results were quite good relative to previous studies. According to Harmatuck, such an approach "avoid(s) biases found in single output cost specification, as well as in those multiple outputs specifications which treat multiple outputs as qualitative variables of a single output index rather than as separate and distinct [page 148]." However, the validity of aggregating inputs in activity sets was not ascertained. Moreover, the quality of the results may well be due to the extremely homogeneous sample of motor carrier firms, as opposed to actual methodological improvement.

A direct comparison of the results provided by the models discussed is not possible. Not only were different samples used, but the *a priori* assumptions made for specification and estimation were also varied.

As outlined by Friedlaender and Spady [1981], and summarized by Winston [1985], several crucial factors must guide the specification and estimation of transportation

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[23] Harmatuck follows the approach proposed by Caves, Christensen and Tretheway [1980] in which a Box-Cox metric is applied to the output variables, allowing the inclusion of zero output levels. This approach is appropriate in the case of multiproduct specification.

cost functions. Basically, there appear to be three fundamental problems. First, and perhaps most important, is the multidimensional nature of the transportation firm's output. Outputs vary by service type (e.g., truckload vs. less-than-truckload), by location (e.g., origin destination pair), and by quality (e.g., speed). For practical reasons, most studies have used a single output measure, such as ton-miles, or reduced multiple outputs to a single dimension which incorporates characteristics such as the number of shipments, shipment size, and length of haul (i.e., a hedonic output measure). Yet, it is well understood that output mix and mode of production have a profound effect on cost.

Second, the varied nature of transportation output complicates the specification of cost functions. Previous cost studies have not used a flexible form joint cost function approach. Chow [1978], Klem [1978], and Koenker [1977] used restrictive single output Cobb-Douglas specifications with second order output terms. Factor prices are omitted from these specifications.[24] Spady and Friedlaender [1978], Cherry [1978], and Keaton [1978] adopted translog cost models, but placed arbitrary restrictions on the nature of output. Cherry used a multiple output translog formulation, but his approach maintained separability among factor prices and output qualities. Only if the cost structure were additively separable could each output be treated separately. But, as stated by Hall [1973], " (additive separability) requires that the technology be nonjoint, so it rules out interaction among the productive process except through the primary factors [page 889]." Since joint production is highly probable for transportation services, a more general functional form is required. In Hall's words, "separability of the transformation function and nonjointness of the technology (should be) available as parametric restrictions (so that they) may be tested with the usual methods of statistical inference [page 889]."

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[24] For convenience, cost function specifications often omit factor prices, assuming that all firms face the same set of prices. This is unlikely to be true if the analysis pertains to different segments of the trucking industry. Moreover, structural characteristics such as input-output separability cannot be assessed if factor prices are not included in the model specification.

The third issue involves long-run versus short-run costs. As Friedlaender and Spady [1981, page 17] have argued,

> "To the extent that regulatory or other constraints prevent the firms in each mode from making optimal adjustments in capacity, they are not generally in a position of long run equilibrium, operating along their long-run cost function. Consequently, efforts to estimate long-run cost functions directly from cross-sectional data may yield seriously biased coefficients and biased measures of marginal costs. This implies that short-run cost functions should be estimated when it is suspected that an industry may be in long-run disequilibrium with chronic excess of capacity."

Long-run cost functions may be derived from correctly specified short-run functions, provided that other relevant technological factors, such as the nature of the route network, are included.[25] However, an adequate range of the required variables is often not present in cross-sectional data sets.

In summary, the estimation of cost functions of transportation industries should incorporate the multidimensional nature of the output as well as shipment characteristics into a sufficiently flexible functional form to permit testing a number of hypotheses concerning the separability, homogeneity, and jointness of the underlying production structure. Also, if constraints preventing optimal capacity adjustment are likely to be present, then a short-run variable cost function should be specified. Finally, in order to properly discriminate behavioral differences, the cost function should incorporate technological factors that may influence costs.

1.3 FRAMEWORK OF ANALYSIS

The theoretical basis for the modelling approach developed in this study is that the nature of trucking does not allow a simple and homogeneous representation of its technological behavior. As was seen in Section 1.1, *trucking* is merely a name describing a collection of

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[25] Chiang and Friedlaender [1984] estimated a multiproduct cost function for the regulated trucking industry that utilizes measures of network connectivity and density as arguments in the cost function.

economic and technical relationships with a number of superficially common characteristics. Such characteristics allow them to be grouped together under the same name at a macroscopic level, but at the micro level, these relationships acquire distinct significance. The proposed model, therefore, emphasizes the heterogeneity of motor carrier technology.

Reviewing some of the econometric studies recently applied to the U.S. motor carrier industry, three major problem areas are brought to attention: model specification, selection of the model functional form, and quality and availability of data. Although these problems are not particular to trucking studies, their intensity is amplified given the particularities and complexities of the industry. While specification relates to the *a priori* knowledge of how the economic and technical relationships defining the structure of production should be taken into account, given the objective of the empirical work, and the fundamental nature of the selection of functional form for the proper characterization of such relationships, the most critical problem is the availability and quality of data.

None of the agencies in charge of collecting information about the freight market in a systematic way , either in the United States or in Brazil, seems able to place the necessary emphasis on the type of data required for this kind of empirical analysis. That is, they do not gather information reflecting the environment in which production of freight transportation takes place. The reasons vary from the lack of interest or resources to the difficulty in obtaining and synthesizing information given the complexity of the industry. No matter what the reasons, the fact remains that empirical studies of this nature are bound by data quality.

In order to overcome data limitations preventing the proper characterization of trucking subtechnologies and therefore, of the implied technical differences, the model developed in this study applies a cluster analytic procedure to identify groups of trucking firms that are similar with respect to technical behavior. The basic assumption is that the unobservable market characteristics and related carrier's operating attributes are reflected in the level of usage of each production factor relative to the others. In other words, clustering is used as an instrument of identifying homogeneous groups of firms based upon their similarity across cost shares.

The focus of the present analysis is the liquid bulk transport segment of the Brazilian trucking industry because of its relatively consistent shipment characteristics in comparison with general commodity transport. Extending the traditional capital-labor-energy-material aggregation, the analysis is carried out with thirteen production factors derived from the IBGE survey for 1981, in order to capture the interactions, at a less aggregate level, of the different types of capital, labor, and other inputs in the production of transportation services.

The significance of the cluster structures is evaluated through the estimation of cost functions specified in such a way to permit not only the testing for technical differences between clusters, but also for homotheticity and homogeneity of the structure of production as well.

1.4 OUTLINE OF THE STUDY

The methodological tools necessary to fulfill the objectives outlined in this chapter are introduced in Chapter II. Given the inherent problems in the specification of cost functions for transportation industries discussed in Section 1.2, the performance of flexible functional forms in modelling the structure of production is the subject of discussion. The objective is to provide some insight surrounding the methodological issues involved in the specification and estimation of such forms, and on their implications when extended to economic aggregation, whose theory is also addressed.

Given the limitations of the IBGE data base, a description of which is presented in Chapter III, the analytical framework used to qualify trucking operations is developed in Chapter IV. In addition to a brief overview of clustering methods, the classification technique used to assess the differences among clusters is introduced. This is followed by the presentation and analysis of the results.

The hypothesis that the clustering-determined structures are associated with distinct production structures is tested in Chapter V. Two sets of cost models are developed. One set explores the appropriateness of pooling firms from different clusters into a single group and a second set of treats each cluster separately. The implications of the estimation results are discussed extensively.

Finally, Chapter VI draws some conclusions based on the analytical results and addresses the direction of future research in this area.

Chapter II

THEORETICAL CONCEPTS

In Chapter I, existing studies of the structure of production of the motor carrier industry were reviewed and critiqued. Problems regarding the specification of cost functions for this type of industry were identified, leading to the conclusion that cost models should be based on sufficiently flexible forms so that a number of hypotheses concerning the nature of the production structure can be modeled and tested.

This chapter will focus on such flexible forms, and on the related theoretical developments and conceptual problems. The objective is to first introduce the methodological tools that will be used, and then to address the limitations inherent in these tools. In Section 2.1, an overview of flexible functional forms is presented. Section 2.2 addresses the theory of economic aggregation, its role and limitations. The main aspects of this chapter are summarized in Section 2.3.

2.1 FLEXIBLE FUNCTIONAL FORMS

Conceptual advances based on the theory of duality between production and cost (profit) functions have led to substantial improvements in empirical cost studies. Under duality, the behavioral assumption of profit maximization subject to a known set of technological constraints implies that the cost function embodies the same technological information as the production function. The emergence of flexible functional forms, based on the work of Diewert [1971, 1973, 1974a], Christensen, Jorgenson, and Lau [1973, 1975], and Lau [1974], among others, supports the application of duality theory to more disaggregated analyses of cost structure than was possible under earlier approaches.

This section begins with a discussion of flexible functional forms. Their advantages and disadvantages are discussed in view of the objectives of this work. The translog cost function is briefly presented, followed by a derivation of the elasticities of substitution and a discussion of methods of avoiding estimation biases caused by neutral and non-neutral efficiency differences. Finally, a discussion of the applicability of the recent theoretical developments is also presented.

2.1.1 The Performance of Flexible Functions

Until very recently, the econometrics of production has been based upon highly restrictive functional forms, the most commonly used being the Cobb-Douglas and the Constant Elasticity of Substitution (CES) forms. Both models impose non-testable restrictions on the elasticities of substitution among factors of production that are unlikely to be present. In the last few years, a large body of literature has arisen concerning the so-called *flexible* functional forms. These functional forms provide a second-order local approximation to an arbitrary twice-differentiable function, and are flexible enough so that no *a priori* restrictions are imposed on their first and second derivatives. This flexibility allows the technology being modeled to exhibit an arbitrary set of elasticities of substitution, allowing previously maintained hypotheses to be tested. The generalized Leontief cost function proposed by Diewert,[1] its extension, the generalized linear-generalized Leontief joint cost function[2] introduced by Hall [1973], the quadratic mean of order r function, the generalized Cobb-Douglas function, and the transcendental logarithmic (translog) proposed by

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[1] See Hall [1973] for reference.

[2] Also known as the hybrid Diewert multiproduct cost function.

Christensen *et al.* [1973] are all flexible functions.[3] Most of them have extensions that enable the analysis of multiproduct technologies and allow testing for separability, homogeneity, and jointness of the underlying structure of production.

However, because of their approximative nature, these flexible functions are only expected to satisfy regularity conditions within the range of sample observations. This inability to satisfy globally the desired regularity conditions makes it impossible to choose among the available forms on a theoretical basis.[4] The literature has some references in which an assessment of the performance of various flexible functional forms in modelling the production structure is made. Unfortunately, none of these studies provide conclusive evidence about their performance.

For example, the translog, the generalized Leontief, and the generalized Cobb-Douglas are compared in Berndt *et al.* [1977] using postwar Canadian expenditure data. The translog was found better both in terms of its consistency with *a priori* reasoning and on formal Bayesian grounds. Appelbaum [1979] developed a generalized Box-Cox extension that contains the translog, the generalized Leontief, and generalized square rooted quadratic forms as special or limiting cases, and based on 1929-1971 U.S. manufacturing data, found the generalized Leontief and generalized square rooted quadratic to be preferred in the primal and dual representation of technology, respectively. Using a similar generalized Box-Cox form on 1947-1971 U.S. manufacturing data, Berndt and Khaled [1979] were able to reject the generalized square rooted quadratic restriction, but unable to reject the generalized Leontief as a special case of the model. Tests concerning the translog were not conclusive.

Guilkey, Lovell, and Sickles [1983, page 591] criticized this approach of performance evaluation of flexible forms:

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[3] Fuss *et al.* [1978] provides a survey of flexible functional forms in the context of production analysis.

[4] They are similar, to some degree at least, with respect to the econometric sophistication required for their estimation.

> "A difficulty with this empirical approach is that the true technology is unknown. Evaluating the performance of flexible forms on the basis of how well they will fit observed data is useful if interest centers on the data, but may be misleading if interest centers on the functional forms themselves."

They suggested that a better suited approach is to begin with a given technology and examine the performance of various forms in representing that technology. Monte Carlo experiments were performed based on a known technology whose complexity was allowed to vary across experiments and on a data base whose characteristics were held constant across experiments. The translog, the generalized Leontief, and the generalized Cobb-Douglas cost functions were tested regarding a number of issues related to the complexity of the technology and estimation methods.[5]

The results were, in general, favorable to the translog specification. However, as the elasticities of substitution deviated from unity or from one another, the otherwise dependable translog approximation deteriorated noticeably; inferences on the magnitude of elasticities of substitution sometimes became incorrect. With respect to estimation, system estimators performed better than single equation estimators for all three specifications, the translog providing a superior result.[6]

More recently, two other flexible forms were introduced: the minflex Laurent [Barnett *et al.*, 1985] and the Fourier form [Gallant, 1981]. While the minflex Laurent apparently outperforms both the translog and the generalized Leontief, tending to satisfy the regularity conditions over a wider range of sample observations, the Fourier series approach proposed by Gallant is globally flexible. However, very little is known about these two forms in terms of their performance in empirical applications.

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[5] They have investigated three issues with respect to complexity of the technology: the effect on the tracking ability of a specific functional form of departures from constant returns to scale or from homotheticity; the effect of deviations from unity, or from one another, in the partial elasticities of substitution; and the effect of input complementarity. Also, they investigated the performance of single equation estimators against the system estimators.

[6] The only case in which the system estimators performed badly was when the true elasticities of substitution were small and positive.

As can be seen, none of the results reported in the literature fully justify the adoption of a specific functional form. The translog, however, has been the one adopted in most recent empirical applications of economic theory. This choice could be explained more by its computational tractability, if compared with other functional forms, than by its performance based on theoretical grounds. Not only does the translog have the least number of parameters to be estimated, but it is the form that allows testing of the largest number of behavioral hypotheses. For example, the generalized Leontief imposes constant returns to scale in the relationship between costs and output levels. Flexibility in scale economies could be obtained, but at a cost of a large increase in the number of parameters. With respect to the quadratic cost function, homogeneity in prices cannot be imposed without loss in the flexibility of the form.

Another aspect of flexible forms is that they are not self-dual, and the decision whether to center the analysis on the primal problem (production function) or on the dual problem (cost function) becomes a choice between two different representations of the technology.[7] However, as summarized by Binswanger [1974a], the use of a cost function rather than a production function for estimating production parameters has several advantages:

- Fewer unrealistic assumptions about the production process are required. Available tools for parameter estimation limit the range of feasible functional forms. This poses a much lesser problem for cost analyses than for production analyses because, regardless of the exact form of the production technology, the cost function can be expected to exhibit certain regularities that are consistent with simplified functional forms.[8]

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[7] Burgess [1975] compared the inferences with respect to substitution possibilities obtained by specifying a cost function and a production function on the same data set. He found very different results even when making the assumption that both models are each approximations of the true technology.

[8] For example, cost functions are homogeneous in prices regardless of homogeneity properties of the production function because doubling of all prices will double the cost but will not affect factor ratios.

- Estimation equations use prices rather than factor quantities as independent variables. This is significant because firms make decisions based on factor prices; hence, factor quantities are endogenous variables.

- In estimating production functions, multicollinearity among input variables is often a problem. This is not the case when estimating cost functions, since multicollinearity among factor prices would be unusual.

- In the derivation of elasticities of substitution or of factor demand, the matrix of estimates for a production function has to be inverted, which may exaggerate errors. No inversion is necessary with a cost function.

Again, it is apparent that the choice among the primal or dual specifications is driven by practical reasons rather than theoretical ones.

Given the above discussion, the translog form was the specification of choice for this work. Its general form, estimation conditions, and the derived economic relationships are discussed in detail in the following section.

2.1.2 The Translog Joint Cost Function

Let the technology be represented by a transformation function $t(y,x)$, where y is a m-dimensional vector of output levels and x is a n-dimensional vector of factor levels.[9] Then there exists a unique cost function $C[y,w]$ which is nondecreasing, positive, linear homogeneous, concave, and differentiable in the price vector w, defined by

$$C[y,w] = \min_x \{w'x : t(y,x) \geq 0\}, \qquad [2.1]$$

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[9] The transformation function $t(y,x)$ is defined and continuous for all nonnegative y and x, and the set $V(y) = \{x: t(y,x) \geq 0\}$, which defines the input bundles that can produce y is closed and strictly convex.

which assigns the minimum cost of producing the vector of outputs y given factor prices w, and fully characterizes the technology defined by $t(y,x)$.

The translog approximation to the cost function $C[y,w]$ can be written as [10]

$$\begin{aligned}\ln C[y,w] &= \alpha_0 + \Sigma_i\, \alpha_i \ln y_i + 1/2\Sigma_i\Sigma_j\, \delta_{ij} \ln y_i \ln y_j + \Sigma_k\, \beta_k \ln w_k + \\ &\quad + 1/2\Sigma_k\Sigma_l\, \gamma_{kl} \ln w_k \ln w_l + \Sigma_i\Sigma_k\, \rho_{ik} \ln y_i \ln w_k\end{aligned} \qquad [2.2]$$

with the symmetry conditions:

$$\begin{aligned}\delta_{ij} &= \delta_{ji}, && \forall\ i,j = 1,\dots,m\\ \gamma_{kl} &= \gamma_{lk}, && \forall\ k,l = 1,\dots,n.\end{aligned} \qquad [2.3]$$

The cost function is nondecreasing in prices, i.e., $\partial C[y,w]/\partial w_k \geq 0$, since increasing any price cannot lower the total production cost of a given output level. Using Shephard's lemma which equates the firm's conditional factor demand for input k with the derivative of the cost function with respect to factor k price, i.e., $\partial C[y,w]/\partial w_k = x_k(y,w)$, this property of the cost function can be translated in terms of the translog form into:

$$\begin{aligned}\partial \ln C/\partial \ln w_k &= [\partial C/\partial w_k][w_k/C]\\ &= [x_k][w_k/C] = [x_k w_k/\Sigma_l\, x_l w_l]\\ &= S_k.\end{aligned} \qquad [2.4]$$

Therefore, $\partial C[y,w]/\partial w_k \geq 0$ is implied by nonnegative factor shares, $S_k \geq 0$, since prices and cost are always nonnegative.

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[10] The multiproduct cost function is presented here as the general case. Results for the single output and variable cost function cases can be easily derived from this general form.

Linear homogeneity in prices is attained by the parametric restrictions:[11]

$$\begin{aligned} \Sigma_k \beta_k &= 1, \\ \Sigma_l \gamma_{kl} &= \Sigma_k \gamma_{kl} = 0, \\ \Sigma_k \rho_{ik} &= 0, \qquad \text{for } i = 1,\ldots, m. \end{aligned} \qquad [2.5]$$

The cost function [2.2] with the parametric constraints [2.3] and [2.5] may be estimated directly. However, additional information is available which can result in improved efficiency of estimation. The result in [2.4] yields the following n behavioral equations: [12]

$$S_k = \beta_k + \Sigma_l \gamma_{kl} \ln w_l + \Sigma_i \rho_{ik} \ln y_i \qquad \text{for } k = 1,\ldots, n, \qquad [2.6]$$

which are all linear in logarithms and have proper exogenous variables on the right side. The system of equations implied by [2.2] and [2.6], with the parametric restrictions [2.3] and [2.5], are the estimation equations for the translog joint cost function.

2.1.3 The Aspects of Technology

Issues of separability, scale, and substitution are among the basic aspects of technology that are of primary interest in analytic studies of the production process. These issues are essential for assessing the impact of policy instruments. Separability is a crucial issue in

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[11] The theory of cost and production requires that the Hessian matrix of the second derivatives of the cost function with respect to factor prices be negative semidefinite to assure concavity of $C[y,w]$ in factor prices. Since the cost function is linear homogeneous in prices, the Hessian matrix is singular. However, according to Burgess [1974], "concavity will be assured if the principal minors of successive order alternate in sign starting negative."

[12] Under constant returns to scale and perfect competition, another m behavioral equations can be obtained by noting that marginal cost is equal to price. The m revenue share equations are written as

$$R_i = \partial \ln C / \partial \ln y_i.$$

However, the equality between marginal costs and output prices has to be valid.

production analysis because it implies unchanged behavior of certain economic quantities and decentralization in decisionmaking; scale effects have implications for long-run growth and for the structure of industry, while substitution among factors of production is critical for the behavior of distributive shares when factor proportions vary.

A technology is said to be separable with respect to a partitioning between inputs and outputs if the transformation function can be written as $t(y,x) = -g(y) + f(x)$.[13,14] A necessary and sufficient condition for the technology to be separable is that the cost function be multiplicatively separable: $C[y,w] = s(y)c(w)$. In other words, the cost function can be written as the product of a function in outputs only by another function in factor prices only. Separability in outputs is a testable hypothesis in the translog cost function because the interaction terms ρ_{ik} between output levels and factor prices can be set to zero. Equation [2.2] then can be rewritten as the sum of two functions, one in outputs and another in factor prices, yielding the relationship

$$\begin{aligned} \ln C[y,w] &= g(\ln y) + h(\ln w) \\ &= \ln s(y) + \ln c(w), \end{aligned} \qquad [2.7]$$

which is the same as $C[y,w] = s(y)\ c(w)$, the necessary and sufficient condition for input-output separability of the transformation function.

Although this kind of separability (homothetic production structure) has been a maintained hypothesis in most empirical production and cost studies [Brown *et al.*, 1979], it is very restrictive as it implies that the ratios of marginal costs, $[\partial C/\partial y_i]/[\partial C/\partial y_j]$, are independent of the input prices. For example, in the case of a trucking firm, this would

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[13] Input-output separability implies a homothetic production structure. See Hasenkamp [1976].

[14] Separability is being discussed here in the context of inputs and outputs. If the function in factor prices is separable with respect to a partitioning of the inputs into r subsets, then the transformation function can be written as $t(y,x) = -g(y) + h[u_1(x_1),\ldots, u_r(x_r)]$ and the joint cost function can be rewritten as $C[y,w] = s(y)\ \Phi[p_1(w_1),\ldots, p_r(w_r)]$. Separability with respect to a partition of inputs into subsets is also a major structural property, and because of its importance to aggregation issues, it will be discussed in more detail in Section 2.2.

mean that the carrier would not differentiate types of services in the process of allocating them; the firm would allocate transportation capacity after producing it, exclusively as a function of the received freight by each service. This process can be viewed as a two-stage optimization performed by the firm. In the first stage, the firm minimizes the transportation capacity costs in order to match the demand for services, and in a second stage, the firm acts as a freight revenue maximizer based on its ability to supply those services. Basically, all services are considered homogeneous; the firm in producing its transportation capacity follows a unique production process independently of the type of service.[15] Therefore, when separability is taken *a priori*, it is assumed that firm management does not perceive different production processes when operating in short or long haul, or in full truck load or less than truck load services. Even if a firm is believed to be managerially inefficient, it is very unlikely that this separability will be verified in transportation. Hence, most transportation cost studies using separability as a maintained hypothesis must be viewed with a certain degree of skepticism.

Returns to scale are defined as the proportional changes in all outputs resulting from proportional increases in all inputs [Caves *et al.*, 1981]. In terms of a cost function, the degree of returns to scale can be computed by [16]

$$r[y,w] \;=\; [\Sigma_i(\partial \ln C/\partial \ln y_i)]^{-1}, \qquad [2.8]$$

which in the case of a translog cost function becomes

$$r[y,w] \;=\; [\Sigma_i(\alpha_i + \Sigma_j\, \delta_{ij} \ln y_j + \Sigma_k\, \rho_{ik} \ln w_k)]^{-1}. \qquad [2.9]$$

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[15] Under the hypothesis of input-output separability no more than one production function exists, even in the case of multiproducts. Moreover, if more then one production function exists, the functions will be necessarily identical [Hall, 1973].

[16] In the case of a variable cost function $C[y,w,z]$ the correct equation is

$$[1 - \Sigma_p(\partial \ln C/\partial z_p)]/[\Sigma_i(\partial \ln C/\partial \ln y_i)],$$

where z_p is the p-th fixed factor.

Homogeneity in the structure of production requires that the joint cost function be homogeneous in outputs. The necessary and sufficient conditions for [2.2] to be homogeneous in outputs are translated into the parametric restrictions in [2.10].

$$\begin{aligned} \textstyle\sum_i \delta_{ij} &= 0, \qquad \text{for } j = 1,\ldots, m \\ \textstyle\sum_i \rho_{ik} &= 0, \qquad \text{for } k = 1,\ldots, n \end{aligned} \qquad [2.10]$$

From Equation [2.9] it follows that the degree of homogeneity of the transformation function is $[\sum_i \alpha_i]^{-1}$, and constant returns to scale are present if $\sum_i \alpha_i = 1$.

Substitutability between factors of production is usually measured by the Allen partial elasticities of substitution (AES).[17] As shown by Uzawa [1962], the elasticities can be derived in terms of factor prices directly from the cost function:

$$\sigma_{kl} = \sigma_{lk} = \frac{CC_{kl}}{C_k C_l}, \qquad [2.11]$$

where C_k is the first derivative of C with respect to the k-th factor price, and C_{kl} is the second cross derivative. Substitution between factors k and l occurs if the AES value is positive, while complementarity is indicated by a negative value. In the case of the translog model, the following result is easily derived:

$$\sigma_{kl} = \frac{\gamma_{kl}}{S_k S_l} + \frac{I}{S_l} + 1 \qquad \forall\ k, l = 1,\ldots, n, \qquad [2.12]$$

where I is an indicator function taking the value 0 for $k \neq l$, or -1 for $k = l$.

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[17] The AES are essentially non-normalized own and cross-price elasticities of factor demand:

$$\sigma_{kl} = (\partial \ln x_k / \partial \ln w_l) \,/\, S_l$$

Although the normalized elasticities (conventional price elasticities) have a more straightforward economic interpretation, the elasticities of substitution have a long history of use in economics. They are partial because the demand effect caused by the change in input price is disregarded.

The AES are related to the factor demand price elasticities (η_{kl}), which leads to the expression of price elasticity in terms of the translog coefficients:

$$\eta_{kl} = \sigma_{kl}\, S_l \qquad [2.13]$$

$$\eta_{kl} = \frac{\gamma_{kl}}{S_k} + I + S_l, \qquad \forall\ k, l = 1, \ldots, n\,. \qquad [2.14]$$

Confidence intervals for the true elasticities can be derived by using the asymptotic variances of the estimates.[18]

The Allen partial elasticity is one of many measures of input association. There are other measures of factor substitutability, and the appropriateness of each measure is the concern of current research. For example, Kang *et al.* [1981] developed an alternative measure of elasticity of substitution which they call *full elasticity of substitution* (ξ_{kl}), and showed that σ_{kl} and ξ_{kl} can provide quite different inferences about the magnitude and direction of factor responses due to differences in the events being measured. The full elasticity is only defined for $k \neq l$, and may be expressed in terms of the AES as

$$\xi_{kl} = S_l\,(\sigma_{kl} - \sigma_{ll}). \qquad [2.15]$$

They advocate the use of the ξ_{kl} when comparing results from competing studies since "they are invariant to the separability assumption often made and therefore do not depend on the unestimated excluded characteristics of the function."

The discussion presented above reflects the economic attributes and relationships derived from the cost function representation of the technology. In terms of estimation,

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[18] The asymptotic variance of σ_{kl} is computed as $var(\gamma_{kl})/(S_k S_l)^2$, which assumes that cost shares are nonstochastic. However, in the estimation of the translog model with the share equations, they are assumed to be stochastic. The *delta method* [Kmenta, 1971, page 444] is an alternative means of computing an estimate of the variance of products of random variables, but the validity of this procedure is doubtful [Kopp *et al.*, 1981].

however, the specification will not be complete if it does not take into account potential efficiency differences among observational units. The way in which to account for these in the model specification is the subject of the following section.

2.1.4 Neutral and Non-Neutral Efficiencies

A non-neutral efficiency difference is one that causes the isoquant map to exhibit non-homothetic properties. As previously mentioned, if the cost function is input-output separable, then the primal transformation function is homothetic, which implies that the rate of technical substitution depends upon the ratio in which inputs are used and not on their absolute values. If efficiency differences exist among observational units (e.g., firms in cross-sections, years in time-series), the specification must account for these in order to avoid bias in estimation. As discussed in Binswanger [1974a], it is necessary to distinguish between two kinds of efficiency differences: (a) those that can be functionally related to a variable such as output (scale effects), time (as a proxy for technical change), education, or management; and (b) those that cannot be functionally related to a variable and which arise from past differences in technical change. If cross-sectionally observed entities had different histories of technical change, they would no longer share the same isoquant.

In the first case, equations [2.2] and [2.6] are correctly specified as long as the y_i represent the phenomena which cause efficiency differences (e.g., output level, time, technology, or managerial structure). The y_i have to affect efficiency at constant logarithmic rates, and data on them must be available.

When no data are available for constructing a variable to capture efficiency differences, unbiased γ_{kl} still result if the efficiency effects of the omitted variable are neutral. In this case, all ρ_{ik} are then null and equations [2.6] are properly specified without data on y_i . However since the α_i are not null, equation [2.2] is no longer correctly specified; the γ_{kl} must be estimated using share equations only.

The second type of efficiency differences can be handled in the same way if the efficiency differences are neutral. Otherwise, it would be necessary to construct an efficiency index and to include it as a variable in [2.6]. If an index were not available, but cross-sectional units could be assigned to groups without internal non-neutral differences, then group dummies in [2.6] would ensure unbiased estimates of the cost function parameters by allowing groups to have differing shares at equal factor prices.[19]

2.1.5 Comments

As mentioned earlier in this chapter, the regularity conditions of the translog approximation can only be extended to the underlying function in the neighborhood of the point of approximation. Some conditions, however, will hold globally, as is the case of homogeneity in prices. Others, like concavity, will not hold globally for any translog function; that is, no parametric restrictions will ensure a Hessian which is globally negative semidefinite. Since concavity is a major requirement for any *well-behaved* neoclassical cost function, one cannot take a translog cost function as an exact representation of a cost function in the feasible range of its arguments.

Thus, in addition to the approximation interpretation of the results, another concern is how to characterize violations of the fundamental properties at points other than that of approximation. Furthermore, even in the case where these properties are satisfied in the sample range, whether this indicates a good approximation is still uncertain.

While Guilkey *et al.* [1981] focused on the complexity of the technology, the studies by Wales [1977] and Caves and Christensen [1980] tried to address these issues by focusing on the range of data points over which translog and generalized Leontief forms provided an adequate approximation to a given technology. Caves and Christensen found, in the case of the translog form, that the regularity properties are not violated over a wider

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[19] This approach was used, for example, by Caves *et al.* [1981] in their analysis of productivity growth in U.S. railroads.

range of data points if the true elasticities of substitution have similar values close to one. With respect to the translog, Wales' simulations indicate that when monotonicity and concavity hold for a large number of sample cases, the elasticities estimates are very good. Moreover, Wales finds that if violations occur for a large number of points, the only conclusion is that of a poor approximation, as opposed to the lack of an optimization process.

The purpose of the discussion presented in this section was to review the methodological issues involved in the specification and estimation of functional forms to model production. These issues are intimately related to the following subject of economic aggregation.

2.2 ECONOMIC AGGREGATION

The common practice in most empirical applications of producer or consumer theory has been to use aggregates of the actual microeconomic commodities involved in the decisionmaking process. Such a practice can be justified given the current state of data collection and statistical estimation methodologies, and by a variety of other issues dependent upon the nature of the analysis undertaken. As Denny [20] states, "it is impossible to imagine economic data without aggregation."

This section summarizes specific concepts related to the broader problem of constructing a consistent measure of commodity aggregates. The literature is so extensive, technically complex, and fraught with controversy, that it can be summarized only in this context. Since the traditional method for aggregating individual inputs (or outputs) is the use of an index number, the theory behind index numbers is presented. Questions are raised regarding not only the implications of applying this theory, but also about the

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[20] See Usher [1980], page 528.

implications that arise from the choice of procedures used for obtaining such indices, and about the methods with which to assess the magnitude of these implications.

2.2.1 The Theory of Index Numbers

Following the work of Barnett [1984], the literature surrounding the construction of price and quantity aggregates can be divided in two major groups: statistical index number theory and economic aggregation theory. Although both theories have the same objective — the definition of reliable means of constructing price and quantity aggregates — only recently have they been brought together in a single framework [Diewert, 1976]. While the object of statistical index number theory is to provide estimators for the ratio of unknown exact aggregates, the object of economic aggregation theory is the derivation of exact economic quantity and price aggregates by providing the conditions for the existence of a true economic aggregate.

Index Numbers

In broad terms, an index number formula is a function of price and quantity information for two entities (individuals, cases, firms, periods, etc.) that indicates, based solely on these data, whether there is any difference in the aggregate consumption or price. In other words, if x is the vector of quantities consumed of the set of goods over which an aggregate is sought, and w is the corresponding price vector, then an index number in the statistical sense is a function, $h(w_1,x_1:w_0,x_0)$, of prices and quantities such that, if a price index is sought, then h should satisfactorily approximate the ratio $P[w_1]/P[w_0]$ of the correct price aggregate $P[w]$ between case 1 and reference case 0. In the case of a quantity index, h should approximate $Q[x_1]/Q[x_0]$, where $Q[x]$ is the correct quantity aggregate.

The most frequently referenced work in this area is that of Fisher's, in which a large number of index formulæ were evaluated according to a set of properties known as Fisherian tests.[21] These tests, or desirable properties, include:

a. the *factor reversal test*: the product of the quantity index times the price index should equal the expenditure ratio between the units;

b. the *commodity reversal test*: an index should be invariant to changes in the ordering of the commodities;

c. the *commensurability test*: an index should be invariant to changes in units of measurement;

d. the *determinateness test*: an index should not become zero, infinite or indeterminate if a commodity price or quantity becomes zero;

e. the *proportionality test*: if all component prices (or quantities) increase by the same factor, then the price (or quantity) index should increase by that factor, i.e., the function h is linear homogeneous;

f. the *point reversal test*:

$$h(w_1,x_1:w_0,x_0) \times h(w_0,x_0:w_1,x_1) \equiv 1 \ ;$$

g. the *identity test*:

$$h(w_0,x_0:w_0,x_0) \equiv 1; \text{ and,}$$

h. the *circularity test*: which requires path independence,

$$h(w_1,x_1:w_0,x_0) \times h(w_2,x_2:w_1,x_1) \ \equiv \ h(w_2,x_2:w_0,x_0) \ .$$

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[21] See Diewert [1976] for reference.

The index formula that satisfied the largest number of tests became known as the Fisher Ideal index, and its price, p_f, and quantity, q_f, formulæ are expressed as:

$$q_f = \left[\frac{w_1'x_1}{w_1'x_0} \times \frac{w_0'x_1}{w_0'x_0}\right]^{1/2} \qquad [2.16]$$

$$p_f = \left[\frac{w_1'x_1}{w_0'x_1} \times \frac{w_1'x_0}{w_0'x_0}\right]^{1/2}. \qquad [2.17]$$

Another index that has many of the Fisherian properties is the Törnqvist-Theil discrete approximation to the Divisia index:[22]

$$q_t = \Pi_i \, [x_i^1 / x_i^0]^{1/2[s_i^1 \,+\, s_i^0]} \qquad [2.18]$$

$$p_t = \Pi_i \, [w_i^1 / w_i^0]^{1/2[s_i^1 \,+\, s_i^0]} \qquad [2.19]$$

where $s_i = w_i x_i / \Sigma_i \, w_i x_i$ is the expenditure share of the i-th component in the aggregate.

Other index numbers in widespread use are the Laspeyres and Paasche formulæ. The Laspeyres and Paasche price indices are expressed as

$$p_l = \frac{w_1'x_0}{w_0'x_0} \qquad [2.20]$$

$$p_p = \frac{w_1'x_1}{w_0'x_1}, \qquad [2.21]$$

respectively, and their quantity indices are defined similarly by interchanging quantities and prices in the above formulæ. As can be seen, the Fisher index is the geometric mean of the Paasche and Laspeyres indices.

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[22] Named after Törnqvist and Theil, the first to recommend its application [Christensen *et al.*, 1979].

The selection of an appropriate formula among these that will adequately express aggregate behavior depends not only on their statistical properties but also on their interpretability. For example, the Laspeyres quantity (price) index shows how much of the change in value of total input results from changes in quantity (prices), since prices (quantities) are held fixed at their reference case levels. The Tömqvist-Theil quantity index also has an easily interpretable functional form, whereas the Fisher Ideal index does not. Taking the logarithm of q_t,

$$\begin{aligned} \ln q_t &= \ln Q[x_1] - \ln Q[x_0] \\ &= \Sigma_i \bar{s}_i \, [\ln x_i^1 - \ln x_i^0], \text{ where } \bar{s}_i = 1/2 \, [s_i^1 + s_i^0] \qquad [2.22] \end{aligned}$$

it is noted that the log change of the aggregate is the share-weighted average of the log changes of the component consumptions. On the other hand, as described by Barnett [1984], "... Fisher Ideal index is a complicated geometric mean of two weighted averages; therefore, changes in the Fisher Ideal index can be difficult to explain to policymakers and difficult to trace to underlying changes in individual components."

However, with recent developments in economic aggregation theory and its conceptual convergence with the theory of index numbers, the choice among index formulæ has become much more dependent upon their economic and econometric attributes than on their Fisherian properties or interpretational advantages.

Aggregator Functions

According to Diewert [1974b] two methods justifying the use of economic aggregates have been suggested. One was developed by Hicks,[23] who showed that "if prices of a group of

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[23] See Diewert [1974b], page 1.

goods change in the same proportion, that group of goods behaves just as if it were a single commodity." The other method, attributed to Shephard,[24] is based on the concept of *homogeneous weak separability*. While the Hicks' price proportionality method refers to the possibility of aggregation, Shephard's factorability condition allows the economy's structure to be written as a composite function of the quantity aggregator function Q and under duality theory, the price aggregator function P.

The concept of separability was conceived independently by Leontief and Sono.[25] A group of variables was said to be separable from the remaining variables in a utility (or production) function if the marginal rates of substitution (MRS) between variables in that group are independent of the values outside that group. Both Leontief and Sono noted that separability is equivalent to functional structure; that is, if $x \geq \mathbf{0}_n$ is a nonnegative n-dimensional vector of factor levels to be aggregated, $z \geq \mathbf{0}_m$ is a m-dimensional vector of other factors, and x is homogeneously weakly separable from z, then the microeconomic production (or utility) function F, where $y = F(x,z)$ is output (or utility), can be written as

$$F(x,z) = F'[f(x),z], \qquad [2.23]$$

where F' is a macro production (or utility) function and f is an aggregator function that satisfies the conditions of (1) positivity, (2) linear homogeneity, and (3) concavity. The function f can be seen as a sectoral utility of a composite commodity. If F is a production function, $f(x)$ may be interpreted as an intermediate output, which is then combined with z to produce $F(x,z)$. Separability implies a stagewise optimization of the structure of production; the group of separable factors constitutes a production unit minimizing costs to satisfy the demand defined by the value of the sub-functions f.[26] Therefore, Leontief-Sono

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[24] Ibid.

[25] Blackorby *et al.* [1978].

[26] This can be extended to groups of separable factors: each group will constitute a production unit minimizing costs to satisfy the demand defined by the value of sub-functions fs.

separability provides a basis for commodity or input aggregates, and introduces the concept of aggregator functions.

The notion of separability in the production function may be extended to the cost function. Duality theory allows the structure of production to be completely represented by either its cost function or production function: to the abstract product $u = f(x)$ there is an associated cost. The unit cost function $c(w)$, dual to $f(x)$, represents the price of the aggregate u.[27] In other words, if the function F is homogeneously weakly separable and the functional form for the aggregator function f is known (or the functional form for its unit cost function $c(w)$ is known), then the aggregate u and its price t can be defined as:[28]

$$u \equiv f(x) \qquad \text{or} \qquad u \equiv w'x \,/\, c(w) \qquad [2.24]$$

$$t \equiv w'x \,/\, f(x) \qquad \text{or} \qquad t \equiv c(w). \qquad [2.25]$$

This approach to computing aggregate quantities and prices was used by Friedlaender and Spady [1981] and McRae and Webster [1982], among others, to obtain prices of energy aggregates.

However, this production/unit cost function approach requires and generates much more information than is necessary for most production analyses. The theory of economic index numbers developed by Samuelson and Swamy,[29] and extended by Diewert [1976, 1980], provides an alternative for obtaining the aggregates directly from their components without having to econometrically estimate the aggregator functions.[30] While Samuelson and Swamy dealt with the way certain index formulæ replicate the true index number, Diewert's main accomplishment was the rationalization of certain functional forms for

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[27] Being linear homogeneous in x (f is homothetic), f has a dual cost function that can be written in the separable form $C(u,w) = u \cdot c(w)$, where u is output and $c(w)$ is a unit cost function. Moreover, $c(w)$ satisfies the same regularity conditions as f defined in (1), (2), and (3) above. See footnote [14].

[28] Kim [1984] provides an excellent summary of the derivation of these results.

[29] See Weyant *et al.* [1981] for reference.

[30] Although it provides a short-cut to estimate aggregate values, the trade-off is that information on cross effects is lost.

index numbers with functional forms for the underlying aggregator function. In other words, Diewert has shown that certain index formulæ are exact for certain classes of functional forms.

Exact and Superlative Index Numbers

A quantity index $q[w_1,x_1:w_0,x_0]$ is said to be exact for an aggregator function f if the functional form of f allows $q[w_1,x_1:w_0,x_0]$ to be written as

$$q[w_1,x_1:w_0,x_0] \equiv \frac{u_1}{u_0} \equiv \frac{f(x_1)}{f(x_0)}, \qquad [2.26]$$

and similarly, a price index, $p[w_1,x_1:w_0,x_0]$, is said to be exact for the unit cost function $c(w)$ if the functional form of c allows $p[w_1,x_1:w_0,x_0]$ to be written as

$$p[w_1,x_1:w_0,x_0] \equiv \frac{t_1}{t_0} \equiv \frac{c(w_1)}{c(w_0)}. \qquad [2.27]$$

Thus the quantity index q equals the ratio of the aggregates u_1/u_0, and the price index p equals the ratio of the unit costs (aggregate prices) t_1/t_0, provided q and p are exact for some f. Also, it can be easily shown that the exact indices defined in [2.26] and [2.27] satisfy the value equivalence condition, i.e., Fisher's factor reversal test:

$$\begin{aligned} p[w_1,x_1:w_0,x_0] \times q[w_1,x_1:w_0,x_0] &= \frac{c(w_1)}{c(w_0)} \times \frac{f(x_1)}{f(x_0)} \\ &= \frac{c(w_1)\,u_1}{c(w_0)\,u_0} \qquad [2.28] \\ &= \frac{w_1'x_1}{w_0'x_0}. \end{aligned}$$

Diewert called a quantity (price) index superlative if it is exact for an aggregator (unit cost) function that provides a second order approximation to a twice-differentiable linear homogeneous function. He has argued that superlative index formulæ should be used when aggregating over goods, assuming that there is a homogeneous weakly separable aggregator function, since they correspond to flexible functional forms for aggregator functions. In other words, he favored them because they always provide a close approximation to the unknown exact aggregates of economic theory. Moreover, all elements of the superlative class lead to approximations that are close to each other. Thus, the choice among superlative index formulæ can be viewed as arbitrary.

Also, he showed that if the prices of the group of commodities vary proportionally, the use of such indices will provide aggregates that are consistent with Hick's aggregation rule, even in the absence of a homogeneous weakly separable aggregator function defined over those commodities. Therefore, superlative index formulæ are consistent with both ways of justifying aggregation over goods.[31]

Among the indices that belong to the superlative class, the Törnqvist-Theil and Fisher Ideal are the preferred according to the literature.[32] The Törnqvist-Theil quantity index [2.18] is exact for a homogeneous translog aggregator function, while the Törnqvist-Theil price index [2.19] is exact for the homogeneous translog unit cost function expressed in [2.29] and [2.30], respectively:

$$\ln f(x) = a_0 + \Sigma_i a_i \ln x_i + \Sigma_i \Sigma_j b_{ij} \ln x_i \ln x_j \qquad [2.29]$$

where $\Sigma_i a_i = 1$, $b_{ij} = b_{ji}$, and $\Sigma_j b_{ij} = 0$, and

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[31] Aggregation over goods is emphasized as opposed to aggregation over sectors or time, since each dimension has its own idiosyncrasies.

[32] The superlative class of index formulæ not only provides a high quality approximation to the exact aggregates, but also possesses many of the Fisherian properties previously described.

$$\ln c(w) = \alpha_0 + \Sigma_i\, \alpha_i \ln w_i + \Sigma_i\Sigma_j\, \beta_{ij} \ln w_i \ln w_j \qquad [2.30]$$

where $\Sigma_i\, \alpha_i = 1$, $\beta_{ij} = \beta_{ji}$, and $\Sigma_j\, \beta_{ij} = 0$.

Because the translog is not self-dual, the factor reversal test is not satisfied. Therefore, two sets of indices can be defined: (1) the pair $\{p_t, \tilde{q}_t\}$, where $\tilde{q}_t$ is implicitly obtained by using the identity [2.28] given p_t, and (2) the pair $\{\tilde{p}_t, q_t\}$, where $\tilde{p}_t$ is also obtained by the value equivalence condition in [2.28] given q_t. These sets have been widely used in empirical research such as that by Burgess [1974], Berndt and Christensen [1973, 1974], Berndt and Wood [1975], and more recently by Weyant *et al.* [1981].[33]

The Fisher Ideal index formulae are exact for a special case of the quadratic mean of order *r* functional form.[34] The aggregator function is defined as

$$f_r(x) = \left[\Sigma_i\, \Sigma_j\, a_{ij}\, x_i^{r/2}\, x_j^{r/2}\right]^{1/r}, \qquad \text{where } a_{ij} = a_{ji}. \qquad [2.31]$$

Similarly, the quadratic mean of order *r* unit cost function is written for $r \neq 0$ as

$$c_r(w) = \left[\Sigma_i\, \Sigma_j\, \alpha_{ij}\, w_i^{r/2}\, w_j^{r/2}\right]^{1/r}, \quad \text{where } \alpha_{ij} = \alpha_{ji}. \qquad [2.32]$$

The quantity index $\boldsymbol{q}_r$ and price index $\boldsymbol{p}_r$ defined in [2.33] and [2.34] are exact for the quadratic mean of order *r* aggregator function and unit cost function, respectively:[35]

$$q_r = \left[\Sigma_i\, (x_i^1/x_i^0)^{r/2} s_i^0\right]^{1/r} \times \left[\Sigma_i\, (x_i^1/x_i^0)^{-r/2} s_i^1\right]^{-1/r} \qquad [2.33]$$

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[33] Although these two pairs have been used interchangeably, some advocate the use of p_t and its implicitly defined quantity index, $\tilde{q}_t$, based on the fact that as the disaggregation level increases, components of the vectors x_1 and x_0 will tend to become zero, making q_t indeterminate. Since prices are always positive, p_t will be defined independently of the level of disaggregation.

[34] Proposed by Michael Denny [Diewert, 1974b].

[35] Theorems 3.8 and 3.10 in Diewert [1974b], pp 35-36.

$$p_r = \left[\Sigma_i \,(w_i^1/w_i^0)^{r/2} s_i^0\right]^{1/r} \times \left[\Sigma_i \,(w_i^1/w_i^0)^{-r/2} s_i^1\right]^{-1/r}, \qquad [2.34]$$

where $s_i = w_i x_i / \Sigma_i w_i x_i$ is the expenditure share of the i-th component in the aggregate. Since this functional form is flexible, these are also superlative indices.

When $r = 2$, equation [2.31] becomes the homogeneous quadratic aggregator function, and [2.32] the homogeneous quadratic unit cost function which are the functions for which the Fisher Ideal indices are exact. [36]

The indices q_r and p_r satisfy Fisher's tests (b) through (g). They do not meet the requirement of path independence (h),[37] and because the related functional form is not self-dual, the value equivalence condition (a) is generally not met as well. Thus, as in the case of the Törnqvist-Theil indices, two pairs of indices can be implicitly defined that satisfy condition [2.28], $\{p_r, \tilde{q}_r\}$ and $\{\tilde{p}_r, q_r\}$. However, for $r = 2$, the factor reversal test (a) is satisfied, i.e., $p_2 \equiv \tilde{p}_2 \equiv p_f$ and $q_2 \equiv \tilde{q}_2 \equiv q_f$.

Although the superlative indices discussed above lead to similar results, Diewert [1976] recommends the Fisher Ideal indices for empirical use based on the following: (1) their functional simplicity, (2) their consistency with revealed preference theory, and (3) their consistency with both a linear aggregator function (infinite substitutability between the commodities to be aggregated) and a Leontief aggregator function (zero substitutability between the commodities to be aggregated).

2.2.2 Conceptual and Analytical Problems

The theory of economic aggregation, briefly discussed above in the context of commodity aggregation, bases the construction of consistent aggregates on essentially two conditions.

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[36] The limit of [2.31] as r tends to zero is the homogeneous translog aggregator function, and similarly [2.32] tends toward the translog unit cost function as r goes to zero.

[37] Under the assumption that the economic agent is maximizing $f_r(x)$ subject to an expenditure constraint, the circularity test will also be satisfied.

The first is related to the restrictions on functional form based on the weak separability property of the underlying production function. If this condition is present, i.e, if the aggregates are conditional upon the properties of the production function, then regardless of the behavior of the many aspects of the economy, consistency in aggregation is maintained. The second condition refers to Hicks' price proportionality method, whose validity is questionable since price proportionality may be observed for a variety of reasons. Aggregates defined based on this condition are less likely to be stable than those conditioned upon the properties of the production function.[38]

However, a practical difficulty with the first approach, based on weak separability and homotheticity, is that it requires information at the most elementary level to test the aggregation conditions. This information is usually unavailable, and very likely to remain so. Thus, as paradoxical as it may seem, aggregates based upon constancy of relative prices are usually a primary requirement in the derivation of structural aggregates. Of course, even when the conditions for additional aggregation are satisfied, there is no guarantee that this result will always be true.

A solution to this problem is not currently available, and may be a long time in coming. Recommendations regarding the direction further research in this area should take have been made. For example, Burmeister [39] hopes that "some approximation theorems can be proved that would indicate error bounds on aggregate production function predictions for certain microeconomic structures." But this has yet to be achieved.

2.3 SUMMARY

The theoretical developments discussed in the previous sections are somewhat disappointing. The intent of flexible forms is to permit testing of arbitrary hypotheses about

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[38] Brown [1981] called the first condition *structural* aggregation, and the second *nominalistic*, since the resulting aggregates are groupings in name only.

[39] In Usher [1981], page 427.

the underlying elasticities. None of the evidence, from either empirical applications to specific data sets or simulation studies, tends to support this purpose. The choice among flexible forms depends heavily on prior knowledge about the elasticities, as shown by Caves and Christensen [1980]. Also, none of the models exhibit acceptable regional properties. Even when they do, as is the case of the recently proposed Fourier approach, little is known about their performance in applications.

With respect to aggregation, the state of affairs is no better. Not only are all of the problems relative to flexible forms present, but in addition, limitations involving data adequacy and availability are also present.

In conclusion, this chapter had two basic purposes: first, to introduce the methodological tools used in this research; and second, to discuss the limitations of these tools. The limitations, however, do not invalidate the use of such theory. On the contrary, they define adequate boundaries within which conclusions may be formulated. One general conclusion that can be drawn is that empirical work will continue to be supported by strongly maintained hypotheses. This, of course, will limit the set of testable hypotheses, since the outcome of a specific test is dependent on both the validity of the hypothesis under examination and the validity of the maintained hypotheses, as pointed out by Fuss *et al.* [1978].

Chapter III

THE IBGE DATA SET

Someone once compared econometrics to a French recipe instructing how many turns to mix the sauce, how many milligrams of salt and spices to use, and how many milliseconds to cook it at exactly 378.6 degrees. When the *statistical* cook checked for the ingredients, he found that some were unavailable. He then substituted chunks of cantaloupe for the hearts of cactus fruit called for in the recipe, ping-pong balls for turtle eggs, green garment dye for curry, and, for a Chateau Lafitte 1853, a can of turpentine.

Clearly, no matter the degree of sophistication of the econometric techniques available, they are of no use without the data properly reflecting the variables in the underlying hypotheses. In this chapter, a descriptive overview of the major source of data available for this work is provided. The preliminary aggregation of cost items is introduced.

3.1 SURVEY CHARACTERISTICS

The survey *Empresas de Transporte Rodoviário*, introduced in Chapter I, is conducted annually by the Fundação Instituto Brasileiro de Geografia e Estatística (IBGE) and covers all types of organized firms, public and private, whose primary activities are to provide road transportation of passengers and freight for hire. The survey's main objective is the identification of the overall sector's structure.

The survey instrument is a questionnaire completed at firm headquarters. The questionnaire is filled out by the respondent, while its distribution, collection, and the initial verification checks are made by the IBGE agent. Freight forwarders, the owner-operator, ambulance and moving services, private carriers, armored services, and car rental firms are excluded from the surveyed population. The data collected does not reflect any additional activities a surveyed firm may participate in.

The questionnaire is divided into 24 titles, grouping information associated with investments and divestments during the fiscal year, fixed assets, personnel and wages, general and operating expenses, revenues, transportation output, fleet characteristics, taxes paid, and fuel consumption. A detailed description of the items under each title is contained in Appendix A. The information provided is for the entire fiscal year, with the exception of personnel, which refers to all those employed on June 30th, and fixed assets, which were those available on December 31st.[1]

3.2 THE LIQUID BULK TRANSPORT SEGMENT

The analysis developed in this work focuses on the liquid bulk transport segment of the Brazilian motor carrier industry. Although this segment represents only 13.5 percent of the total number of firms in the business,[2] its relatively consistent shipment characteristics in comparison with solid bulk transport, for example, minimize the effect of an incomplete characterization of transportation output.

From the original sample of 1400 carriers in this segment,[3] only 1172 for which there were adequate data were included in the working sample. All 1172 firms met the following criteria:

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[1] IBGE accepts information from firms with fiscal year ending on September 30th of the reference year to March 31st of the next year.

[2] Refer to Table 1.6, in Chapter I.

[3] See footnote 14 of Chapter I.

- information provided reflected the activities from 01/01/81 to 12/31/81;
- the firm had a non-zero total for personnel, wages, general and operating expenditures, transportation output, fleet, fleet capacity, fuel consumption, and fuel expenditures;
- the corresponding salary and wages paid had to be reported for each type of labor if personnel was reported, and vice versa;
- the corresponding expenses had to be reported for each type of fuel if consumption was reported, and vice-versa;
- if fleet size was reported then the corresponding capacity had to be reported for each class of truck, and vice-versa;
- if the value of vehicle capital stock had not been reported, then expenses with rent and leasing of trucks had to be reported, and vice-versa;
- no obvious errors in the data were present (an example of such errors would include one firm with three trucks which reported a total output of 2 tons).

Table 3.1 shows the distribution of these firms within five geographical regions of Brazil. The large concentration of carriers in the eastern and southern regions, representing 78 percent of the sample, can be easily explained by the relatively higher degree of industrialization in these two regions. The percentage of transportation produced (tons) within each region is displayed in Table 3.2.

By comparison with the figures describing the overall sector, it can be seen that the degree of specialization with respect to the type of traffic lines is similar.[4] Over 95 percent of the sample typically operates with no regular lines (type 5), or in interstate (type 300) and intercity (type 2000) traffic lines. Their distribution, however, is different. One-third of the liquid bulk segment operates on regular lines, twice the industry's percentage.

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[4] See Tables 1.5 to 1.7

Table 3.1: Geographical Distribution of Carriers by Operation Type

operation [a]	north	northeast	east	south	central	Brazil
5	11	103	425	207	44	790
40	.	.	1	.	.	1
300	.	10	50	19	19	98
305	.	.	.	.	2	2
2000	4	26	121	61	18	230
2005	.	1	3	2	.	6
2300	1	2	6	7	.	16
2305	.	.	2	1	.	3
2340	.	.	1	.	.	1
10000	2	1	4	2	1	10
10005	.	.	1	.	.	1
10300	.	.	1	.	.	1
12000	.	2	3	1	1	7
12300	.	.	4	1	.	5
12305	.	.	1	.	.	1
Total	18	145	623	301	85	1172

Note: a. regular lines: intracity (10000), intercity (2000), interstate (300), international (40); no regular lines (5).

Table 3.2: Regional Distribution of Output (tons) by Type of Operation (%)

operation	north	northeast	east	south	central	Brazil
5	1.74	5.90	51.98	18.24	6.36	84.23
40	.	.	0.02	.	.	0.02
300	.	0.26	1.52	0.94	0.35	3.07
305	.	.	.	.	0.05	0.05
2000	0.23	0.66	3.79	0.76	0.47	5.90
2005	.	0.02	0.10	0.06	.	0.18
2300	0.24	0.09	0.83	0.88	.	2.05
2305	.	.	0.07	0.85	.	0.93
2340	.	.	0.52	.	.	0.52
10000	0.28	0.01	0.04	0.01	0.01	0.35
10005	.	.	0.05	.	.	0.05
10300	.	.	0.05	.	.	0.05
12000	.	0.01	0.10	0.04	0.06	0.21
12300	.	.	0.46	1.94	.	2.40
12305	.	.	0.02	.	.	0.02
Total	2.48	6.94	59.54	23.74	7.30	100.00

Although average firm level data regarding number of employees and revenue levels for carriers without regular traffic lines are similar to the industry's numbers, carriers with regular lines have surprisingly lower figures. In Table 3.3, the data show that the average labor and revenue levels are almost five times smaller than those in the same category at the industry level (shown in Table 1.4).[5]

Table 3.3: Comparison Between Carriers With and Without Regular Lines

operation	firms	revenues $10^6 US\$$	output 10^6 *tons*	personnel	employees/ firm	revenues/ firm $10^3 US\$$	revenues/ employee $10^3 US\$$
no regular lines	790	315.3	16.8	12029	15.2	399.06	26.21
regular lines	382	62.4	3.2	2735	7.2	163.30	22.81
all	1172	377.7	20.0	14764	12.6	322.22	25.58

3.2.1 Cost Structure

The survey classifies firms' expenditures according to three major groups: general expenditures, including all expenses incurred with an administrative character; operating expenditures, reflecting those directly related to transportation services; and payroll expenditures. All three comprise 31 cost accounts.

The average contribution of each of these accounts within each expenditure group is shown in Table 3.4. Given that not all firms have entries for all accounts, Table 3.4 also shows that contribution averaged within firms reporting non-zero entries for the account.

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[5] Within the liquid bulk segment, firms without regular lines have on average 2.5 times the carrying capacity of firms with regular lines.

Table 3.4: Distribution of Variable Cost Items (%)

cost account	code	share[a]	share[b]	entries
GENERAL EXPENSES				
rent and leasing of land and buildings	GEN01	4.57	15.82	339
rent and leasing of office equipment	GEN02	0.15	13.89	13
maintenance of buildings and equipment	GEN03	0.84	5.41	183
advertising	GEN04	0.52	3.08	197
communications	GEN05	3.68	7.64	564
loans and financing of working capital and fixed assets	GEN06	12.10	23.48	604
office supplies and cleaning material	GEN07	1.61	3.33	565
labor related expenses	GEN08	51.14	51.63	1161
insurance of buildings and equipment	GEN09	1.34	6.29	249
outside services	GEN10	16.46	18.81	1026
utilities (electricity)	GEN11	1.69	4.11	482
miscellaneous	GEN12	5.89	8.64	799
total		100.00		
OPERATING EXPENSES				
vehicle maintenance and parts	OPR01	21.45	21.80	1153
printed matter used in traffic	OPR02	0.19	0.70	327
fuel and lubricants	OPR03	53.66	53.66	1172
outside vehicle maintenance and repair	OPR04	8.12	9.03	1054
terminal fees	OPR05	0.03	1.30	31
licensing	OPR06	2.12	2.16	1149
vehicle insurance	OPR07	1.61	1.86	1012
purchased capacity	OPR08	9.06	35.99	295
brokerage	OPR09	0.13	4.22	35
indemnities	OPR10	0.23	2.42	110
rent and leasing of trucks	OPR11	1.15	11.30	119
rent / leasing of containers and equipments	OPR12	0.01	0.89	19
miscellaneous	OPR13	2.25	3.51	751
total		100.00		
PAYROLL				
owners with activity	SAL01	42.62	44.96	1111
administration	SAL02	5.15	18.03	335
traffic	SAL03	49.80	53.30	1095
maintenance	SAL04	1.84	11.93	181
other	SAL05	0.51	5.60	106
gratuities and profit share	SAL06	0.08	6.19	15
total		100.00		

Note:
a. average share within the 1172 firms.
b. average share within firms with an entry in that account.

The three major components of general expenditures are the financial costs incurred for working capital (GEN06), employers' contributions to unemployment compensation, retirement programs, and other employees' benefits (GEN08), and the cost of outside services (GEN10) like legal, accounting, and data processing services. Not surprisingly, the primary sources of operating expenditures are fuel (OPR03), vehicle maintenance and repair (OPR01 and OPR04), and the cost of purchasing capacity from the independent trucker (OPR08). Owners and traffic personnel account for more than 90 percent of payroll expenses.

The structure of capital assets is given in Table 3.5, with vehicle capital stock representing almost 93 percent of the total value of fixed assets. It is clear, therefore, that this type of trucking activity does not require substantial capital investments aside from vehicles.[6]

Table 3.5: Distribution of Fixed Assets (%)

cost account	code	share[a]	share[b]	entries
land and buildings	NLA01	3.01	19.53	180
machinery and equipments	NLA02	1.45	5.36	318
fixtures	NLA03	0.45	2.63	199
furniture and office equipment	NLA04	1.28	2.86	525
transportation means	NLA05	92.77	92.77	1172
in process	NLA06	0.24	18.64	15
concessional rights	NLA07	0.04	2.99	15
financial interests	NLA08	0.76	4.19	212
total		100.00		

Note: a. average share within the 1172 firms.
b. average share within firms with an entry in that account.

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[6] The large percentage of zero entries could be also related to a lack of proper bookkeeping.

If all capital assets were rented or leased, then the value of rent and leasing payments would equal the value of the service of capital stock. As this is not the case, the cost of owning the various types of fixed assets was assumed to be 14 percent of the declared value of each asset.[7] Figure 3.1 depicts the average contribution of each group of expenses to the firm's total production costs. On average, over 80 percent of the cost is shared by operating costs and payroll.

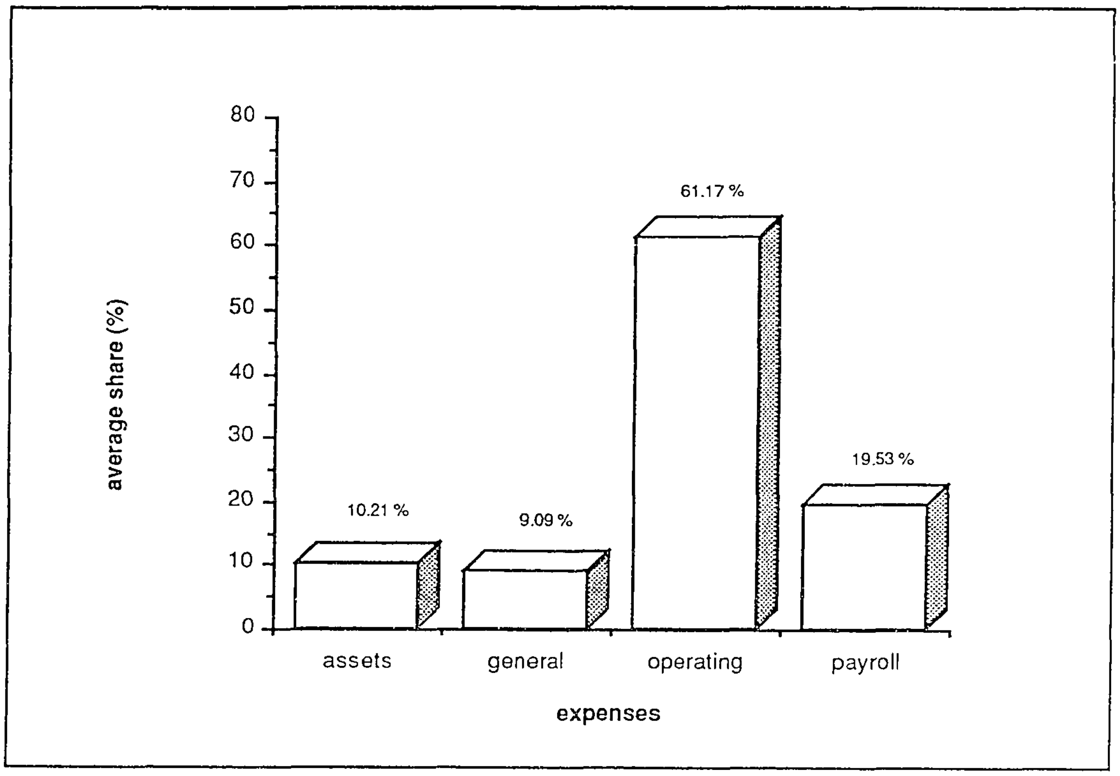

Figure 3.1: Average contribution of each expense to the total cost

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[7] The 14 percent rate is implied by a depreciation and amortization rate of approximately 16.3 percent. This percentage is the average ratio of AMORT/NLA05 within 224 firms having declared amortization and depreciation (AMORT) with transportation means (NLA05) as the only fixed asset.

With respect to fleet composition, straight trucks are the most common vehicles used, followed by the tractor-trailer combination. Table 3.6 contains the number of carriers using a specific type of vehicle according to the categories defined in the IBGE survey. The vast majority of firms operates diesel-fueled trucks, as can be inferred from Table 3.7, which shows the cost breakdown of oil derivatives usage.

Table 3.6: Fleet Profile

vehicle type	code	number of carriers
trucks	FLT11	1054
pickups and vans	FLT12	71
trailers	FLT13	245
piggyback trailers	FLT14	42
tractor	FLT19	245
towing trucks	FLT20	14
other vehicles	FLT21	1

Table 3.7: Expenditures with Oil Derivatives (%)

fuel type	code	share[a]	share[b]	entries
alcohol	ENR11	0.06	4.85	15
gasoline	ENR12	2.37	22.58	123
diesel	ENR13	89.13	89.89	1162
fuel oil	ENR14	0.08	5.28	17
kerosene	ENR15	0.24	4.67	6
gas from oil	ENR16	0.00	0.00	0
other fuels	ENR17	0.05	3.38	18
lubricants	ENR18	8.29	8.86	1097
total	OPR03	100.00		

Note: a. average share within the 1172 firms.
b. average share within firms with an entry in that account.

3.2.2 Preliminary Aggregation

Given the impossibility of working with all cost items, a preliminary aggregation was conducted. The 39 accounts were collapsed into 13 according to three categories: operation, administration, and capital. The aggregates and their components are shown in Table 3.8.

Table 3.8: Aggregate Cost Accounts

class	description	name
operation	MAINTENANCE AND REPAIR	OPR1
	vehicle maintenance and parts	OPR01
	outside vehicle maintenance and repair	OPR04
	lubricants	ENR18
	PURCHASED CAPACITY	OPR2
	purchased capacity	OPR08
	FUEL	OPR3
	alcohol	ENR11
	gasoline	ENR12
	diesel	ENR13
	LABOR IN OPERATION [a]	OPR4
	personnel in traffic	SAL03
	personnel in maintenance	SAL04
	fringe benefits	GEN08
	gratuities and profit share	SAL06
	OTHER EXPENDITURES IN OPERATION	OPR5
	printed matter used in traffic	OPR02
	terminal fees	OPR05
	brokerage	OPR09
	indemnities	OPR10
	rent and leasing of containers and other equipments	OPR12
	miscellaneous	OPR13
	fuel oil	ENR14
	kerosene	ENR15
	gas from oil	ENR16
	other fuels	ENR17

Table 3.8: (continued)

class	description	name
administration	MAINTENANCE OF BUILDINGS AND EQUIPMENT	ADM1
	maintenance of buildings and equipment	GEN03
	insurance of buildings and equipment	GEN09
	FINANCIAL EXPENDITURES	ADM2
	financing of working capital and fixed assets	GEN06
	CONTRACTED SERVICES	ADM3
	outside services	GEN10
	LABOR IN ADMINISTRATION [a]	ADM4
	owners and associates	SAL01
	administration staff	SAL02
	other employees	SAL05
	fringe benefits	GEN08
	gratuities and profit share	SAL06
	OTHER EXPENDITURES IN ADMINISTRATION	ADM5
	advertising	GEN04
	communications	GEN05
	office supplies and cleaning material	GEN07
	utilities	GEN11
	miscellaneous	GEN12
capital[b]	VEHICLE CAPITAL STOCK	KAP1
	transportation means	NLA05
	licensing	OPR06
	insurance	OPR07
	rent and leasing of trucks	OPR11
	EQUIPMENTS	KAP2
	machinery and equipment	NLA02
	fixtures	NLA03
	furniture and office equipment	NLA04
	rent and leasing of office equipment	GEN02
	LAND AND BUILDINGS	KAP3
	land and buildings	NLA01
	rent and leasing of land and buildings	GEN01

Note:
a. benefits (GEN08) and gratuities/profit distribution paid (SAL06) were assigned to each class in proportion to the class payroll.
b. assuming 14 percent of the declared value of each asset.

Under this thirteen factor aggregation scheme, the average contribution of each expenditure to the total cost of production is shown in Table 3.9, where operation accounted for about 70 percent of the cost, with administration and capital sharing, almost equally, the remaining 30 percent.

Table 3.9: Cost Share of Aggregate Accounts

cost account		code	share[a]	share[b]	entries
OPERATION			71.92		
maintenance and repair	OPR1	O1	20.34	20.34	1172
purchased capacity	OPR2	O2	7.17	28.49	295
fuel	OPR3	O3	29.19	29.19	1172
labor in operation	OPR4	O4	13.50	12.50	1172
other expenditures in operation	OPR5	O5	1.71	2.22	892
ADMINISTRATION			14.57		
maintenance of buildings and equip.	ADM1	A1	0.22	0.72	853
financial expenditures	ADM2	A2	1.43	2.78	604
contracted services	ADM3	A3	1.33	1.52	1026
labor in administration	ADM4	A4	10.29	11.24	1073
other expenditures in administration	ADM5	A5	1.30	1.58	967
CAPITAL			13.51		
vehicle capital stock	KAP1	K1	12.43	12.43	1172
equipments	KAP2	K2	0.23	0.46	596
land and buildings	KAP3	K3	0.85	2.14	465

Note: a. average share within the 1172 firms.
b. average share within firms with an entry in that account.

3.3 COMMENTS

Although highly detailed, the IBGE survey is far from being the ideal instrument for a study of this nature. The main problem stems from the fact that the questionnaire is designed to accommodate both passenger and freight firms, when each type of firm has its own set of production factors and particular methods of keeping records. Thus, the accuracy of some of the answers is questionable. Also, while the classification of output by type of lines is adequate for passenger transport firms, this characterization is ambiguous and does not qualify the attributes of freight for freight transport firms. Moreover, passengers and tons transported are incomplete measures of production since the distances involved are not included.

Another problem is the lack of information about the independent trucker. For example, it would be extremely important to know what percentage and type of traffic is actually transferred to the independent trucker. Clearly, this type of information would enhance the knowledge of the role of the independent trucker in the production process.

These are problems that compromise the quality of analysis. A proposed methodology which may help to counter some of the inadequacies inherent in the data is described in the following chapter.

Chapter IV

CLASSIFYING PRODUCTION BEHAVIOR

As previously discussed, in any empirical work of an econometric nature the theoretical relationships among economic variables are always confronted with reality, i.e., with the availability of an observable counterpart of the particular set of theoretical variables. Moreover, since for a given model specification one must assume that each observation in the set was originated from the same parametric model, another problem commonly faced is that of which data to use for estimation and hypotheses testing.

In production studies, for example, the importance of distinguishing among movements along, as well as shifts in, a production function is often emphasized. As addressed in Chapter II, the estimation of a cost function to describe technology involves the *a priori* assumption that the economic agent is efficient, and that if inefficiencies or efficiency differences are present, they must be accounted for by the model specification. Potential technical differences arising because firms may not operate on the same isoquant have important implications for the behavioral assumptions underlying the structure of technology being modeled. It is clear, therefore, that the set of available data has to be subjected to careful analysis, and possibly to some sort of splitting or winnowing, in order to achieve reasonably homogeneous subsets with respect to the parametric model under consideration.

The nature of trucking in general does not allow the assumption of an industry behaving homogeneously according to a single technology. Even when sectors within the industry may be defined according to some common characteristic (e.g., liquid shipments vs. dry goods, TL vs. LTL), the distinguishable markets served within sectors, clearly having distinct attributes and technical opportunities, weaken the assumption of an identical technological behavior. The attributes of the data set used in this work do not explicitly allow the characterization of sectors within sectors. The estimation of a model for the whole industry, or for a particular sector, without consideration for the possible differences in production structure, would inevitably be a misrepresentation of the underlying technological structure.

In the following sections a procedure for sample splitting using cluster analysis techniques is presented. The basic assumption is that unobservable market and related firm operating attributes are implicit in the distribution of firms' cost shares. In other words, the technology and market constraints faced by the firm — the environment in which production takes place — are reflected in the level of usage of each production factor relative to the others. Firms having similar factor cost share profiles are assumed to have similar technical structures. Cluster analysis is used as a means of identifying homogeneous groups based upon the similarity across cost shares.

In Section 4.1 the terminology used in the remainder of the chapter is introduced. First, a brief overview of the concepts behind clustering theory is presented. Emphasis is placed on the performance and reliability of existing methods rather than on the underlying theory which is fully described in most basic statistical literature. In addition, a nonparametric technique for classification (CART), used in this analysis to describe the differences between the determined clusters, is discussed.[1]

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[1] CART — *Classification and Regression Trees*, is a nonparametric technique for classification and regression introduced by Breiman *et al.* [1984].

Sections 4.2 and 4.3 present the results of an application of these procedures to a group of trucking firms operating with liquid shipments. Finally, Section 4.4 summarizes and discusses the primary findings of this analysis.

4.1 METHODOLOGY

The primary objective of this chapter is to identify segments within the industry based on similar cost share distributions. In other words, the goal is to uncover groups of trucking firms homogeneous with respect to cost allocation, given the assumption that these groups will be identifiable in terms of the intensity of usage of certain production factors relative to the usage of others.

4.1.1 Clustering Methods

Cluster analysis is a generic name for a variety of quasi-statistical methods for classification that have been developed in several different fields. Although each was developed with a specific theoretical and methodological orientation, these methods have the common purpose of assigning objects into groups suggested by a set of attributes, such that objects in a given group or cluster tend to be similar to each other with respect to some trait, and objects in different clusters tend to be dissimilar. The most popular methods for clustering are single linkage, complete linkage, average linkage, centroid, and Ward's minimum variance.[2]

One of the major problems inherent in cluster analysis involves the selection of the principle used to place similar objects into clusters.[3] The large number of clustering procedures available makes any generalization of the techniques very difficult, and the lack of a well-articulated and solid theoretical structure supporting cluster analytic methods

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[2] Their performance, however, cannot be inferred from their popularity.

[3] Blashfield and Aldenderfer [1978] present a discussion on the state of the literature on cluster analysis. The article is a first step in the attempt to consolidate the large number of clustering procedures which have been developed in the "wide range of sciences interested in clustering."

makes them heuristic-based procedures. As such, as pointed out in Milligan [1981, page 380], "none of the clustering methods currently in use offer any theoretical proof which ensures that the algorithm will recover the correct structure."

This issue of validation has been the main focus of extensive research in the past few years.[4] Many cluster analytic methods have been evaluated and compared through Monte Carlo experiments with mixed results. In Milligan [1980], for example, the effect of different types of error perturbation on fifteen clustering methods was examined. Hierarchical methods were found to be "differentially sensitive to the type of error perturbation." Comparatively, the average linkage method and Ward's minimum variance method had better overall performance in many of the simulation studies, most of them reviewed in Milligan [1981].

The simulation results, however, have not been consistent across studies. This could be explained by the fact that most techniques tend to find clusters having specific characteristics related to size, shape, or dispersion.[5] Thus, the results were typically biased with respect to the structure of the artificial data sets used. Nevertheless, a conclusion that can be stated is that there is no *best* method for clustering. The recovery ability of the methods currently in use is somewhat data dependent.

Another problem with cluster analysis is the determination of the number of clusters in the final solution. None of the clustering methods provide satisfactory information on the number of partitions in the data. Non-hierarchical methods usually require an *a priori* specification of the number of partitions, while hierarchical procedures generate as many solutions as the number of cases in the data set. Unfortunately, none of the usual parametric or nonparametric significance tests are valid for testing differences between

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[4] Milligan [1980, 1981] and Milligan and Cooper [1986] are among the works published in this area.

[5] For example, average linkage tends to find clusters with equal variances, while K-means and Ward's methods are biased toward clusters with the same size.

clusters. Consequently, a variety of procedures or stopping rules for determining the number of clusters in the data set has been proposed.[6]

In a recent article, Milligan and Cooper [1985] evaluated the performance of thirty procedures for determining the number of clusters. Again, the simulations did not provide conclusive results; some procedures performed quite well given a certain data structure, and not all well with a different data structure. The cubic clustering criterion, the pseudo F statistic, and the pseudo t^2 statistic are among those that performed effectively in an overall sense.[7] Due to their underlying assumptions, none is guaranteed to perform well in all situations. The soundest recommendation is to use several criteria jointly in determining the appropriate number of clusters for a given data set.

More generally, a consensus among results from different techniques should be sought in order to validate an estimated cluster structure. If this consensus is not achieved, no conclusion can be drawn regarding the existence of clusters; only that the methods failed to properly uncover them.

Once a cluster solution is obtained, it is usually subjected to a classification analysis in order to determine which variables are most responsible for the profile differences between the clusters. Although discriminant analysis has generally been the method of choice, CART was the classification technique adopted in this research to assess the differences in cluster profiles. The basic concepts of CART are introduced next.

4.1.2 CART Methodology

The CART methodology, described by Breiman *et al.* [1984], is a recently developed and powerful alternative to traditional parametric methods of classification and regression.[8,9] In

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[6] SAS Institute Inc. [1985a], Chapter 6, briefly discusses many of the proposed criteria.

[7] See Milligan and Cooper [1985] for references regarding these statistics.

[8] For a comparison between CART, discriminant analysis, and logistic regression, see Breiman *et al.* [1984], Komor [1987], and Loh and Vanichsetakul [1986].

[9] Although the algorithms for classification and regression are quite similar, only the classification algorithm will be discussed here.

the case of classification, the method produces a classification rule that can be represented as a binary decision tree, providing, therefore, a better way to visualize the influence of the various variables on the prediction of class membership.

Three points were considered in the selection of CART as the classification procedure. First, CART is nonparametric, therefore no distributional assumption has to be made with respect to the variables used. Second, CART classifiers are quite robust, and can produce unbiased estimates of error rates that are substantially smaller than those obtained by the usual parametric methods. Third, and perhaps most importantly, CART accounts for different associations between variables that may exist in different parts of the data. Consequently, a better understanding of the interactions between the many variables used in the prediction may be achieved.[10]

The CART algorithm can be described as a branching technique through which a large binary tree is grown by successive splits in the data. Subsequently, a pruning algorithm selects a simpler tree with minimal estimated error rate. The tree generation algorithm is summarized below:

- for each variable S_i and all values k within the range of S_i, all splits of the form $S_i \leq k$ are examined; the split giving the best separation is then determined;
- the data are split according to the variable providing the best separation, which originates two other nodes;[11]
- the steps above are repeated for each subsequent node.

The branching process continues until a very large tree containing only a small number of cases in each of the terminal nodes is obtained. Then a sequence of smaller and

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[10] The cost associated with these features is somewhat high. CART requires large data sets to attain stable results, and because it chooses its splits at each node using exhaustive searches, extensive computer resources are necessary.

[11] Two criteria are available to evaluate class separation: Gini and twoing. The choice of one over the other seems to have no significant implications. Both are fully explained in Breiman *et al.* [1984], Chapter 4.

smaller trees is selected by the pruning algorithm such that each subtree in the selected sequence has a lower apparent error rate (node impurity or misclassified cases) than any other subtree of the same size.

The next step involves the selection of the best tree from this sequence. This is achieved by assessing estimates of the true error rate either by a *test set* or by *cross-validation*, and picking that tree with the smallest estimated true error rate.[12]

In analyzing a CART-produced tree, only a few of the many variables may have been used to generate the splits. In order to avoid a possibly erroneous conclusion regarding the predictive power of the variables that were not used, CART provides a measure of variable importance based on the concept of association between splits. Two splits are said to be strongly associated if they generate an equivalent result, i.e., if almost all the cases sent to the next nodes by one split are sent the same way (or in reverse order) by the other split. These *surrogate* splits are then used to construct the measure of variable importance.

4.2 ANALYSIS

This section will review the results of a clustering procedure applied to the set of trucking firms introduced earlier. In view of the present state of cluster analysis, its use must be seen as an exploratory or preclassification instrument to formulate, rather than test, categorizations present in this sector of the trucking industry.

4.2.1 Cluster Profiles

In the production of transportation services, carriers are assumed to follow a technology that relates the flow of output to the service of the thirteen basic inputs listed in Table 3.9.

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[12] The CART literature reports that both cross-validation and test set methods have been found very reliable in simulation results, and the trees selected using them have always been close to optimal trees.

The cost shares associated with these basic inputs define the set of attributes across which similarity among firms is assessed. The squared Euclidean distance computed in this thirteen-dimensional space was taken as the measure of similarity between the 1172 firms in the data set.[13] Since cost shares are proportions, and therefore quantitative and unitless variables, no standardization was required to account for the lack of scale invariance of the Euclidean metric.

Two clustering algorithms were applied in order to validate the results of a particular algorithm: the average linkage and the centroid methods. Ward's minimum variance algorithm was not used because its distributional assumptions were unlikely to be met. Also, it has the strong tendency to generate clusters with the same number of observations.

The results of the average linkage method applied to the data are summarized by the dendograms in Figures 4.1 and 4.2 for the last fifteen clusters joined. Figure 4.1 presents the groupings that have been effected, at each successive level, from fifteen clusters to one cluster. Figure 4.2 displays the dissimilarity level at which grouping takes place. The number of firms in each of the fifteen clusters is shown at the bottom of the tree.

As can be seen in Figure 4.2, there is a relatively large separation among the last two clusters joined. This suggests the existence of at least two sets of firms that are heterogeneous with respect to their cost share profiles. After fifteen partitions are obtained, the average linkage between the last cluster fusion decreases to less than one half of the largest linkage. Thus, while more than two groups can be discriminated, the existence of more than fifteen groups is unlikely. In fact, the values obtained for the pseudo statistics and cubic clustering criterion (CCC), shown in Figure 4.3, indicate no more than twelve significant clusters.[14]

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[13] The Euclidean distance between two vectors **u** and **v** is the norm of the vector difference, i.e., $\|u - v\|$.

[14] The pseudo F statistic and CCC measure the separation between all the clusters at a given level, while the pseudo t^2 measures the separation between the two clusters most recently joined. Significant cluster structures are then indicated by peaks of the pseudo F and CCC matched with a small pseudo t^2 and a large pseudo t^2 for the next two clusters joined.

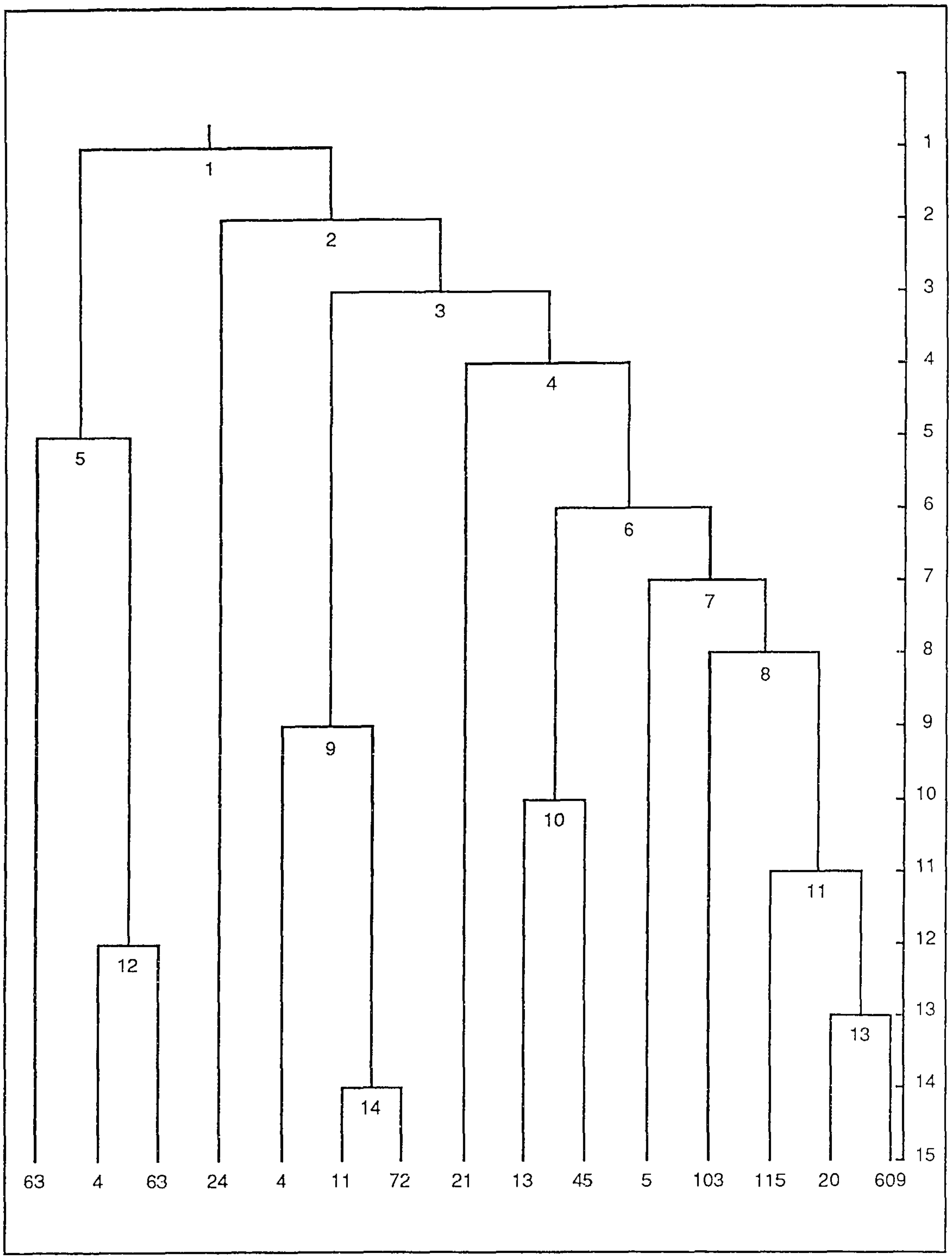

Figure 4.1: Tree structure from average linkage clustering

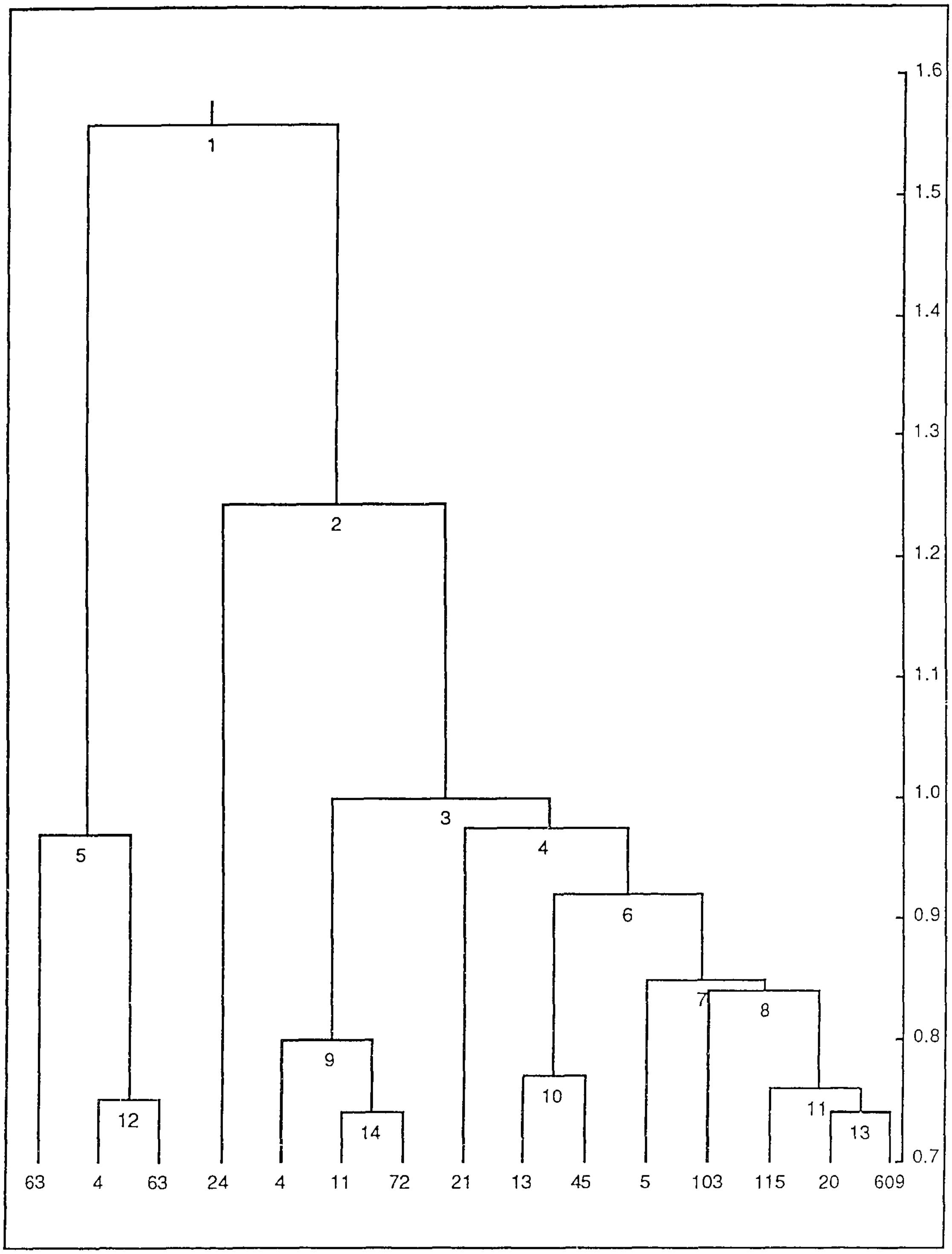

Figure 4.2: Average linkage between clusters joined

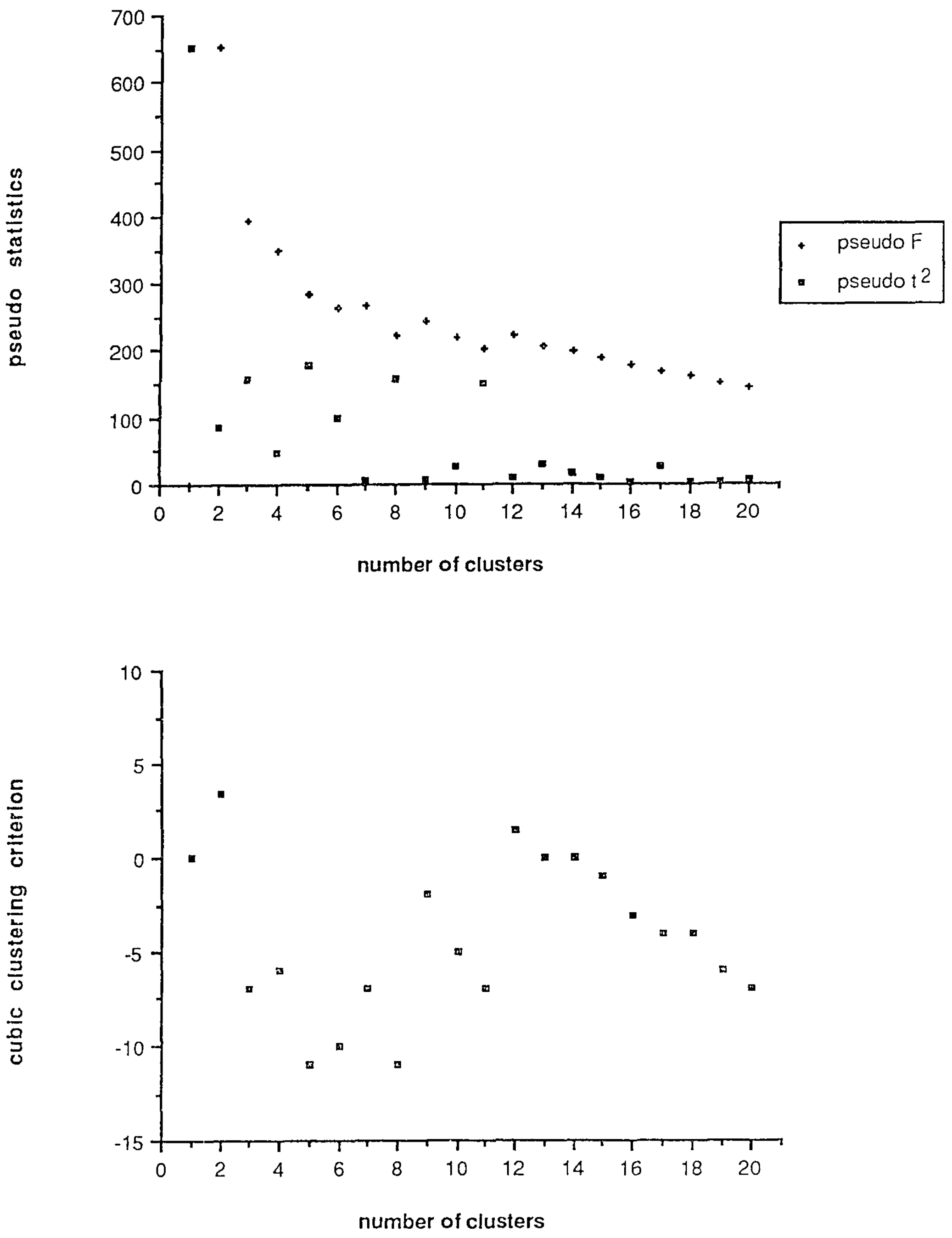

Figure 4.3: The pseudo statistics and the cubic clustering criterion from the average linkage method

The pseudo F statistic peaks at 2, 7, 9, and 12 clusters, dropping steadily after that. The pseudo t^2 drops abruptly from a high value at 2, 4, 7, 9, and 12 clusters. The CCC also peaks sharply at 2 and 12 clusters, with lesser peaks at 4, 7, and 9 clusters. Thus, there is strong evidence for a two partitions solution, with possibly less significant structures at the level of 4, 7, 9, and 12 clusters.

Table 4.1 shows the mean share profile for the sample and for each of the solutions suggested by the procedure. It is notable that, on average, over 90 percent of the total cost is represented by the expenditures on only six inputs: fuel (O3), maintenance and repair (O1), labor in traffic (O4), vehicle capital costs (K1), labor in administration (A4), and hired capacity (O2).

At the two-partition level, while subgroup 2.2 has the same overall mean profile as the full sample, subgroup 2.1 is distinctive with respect to hired capacity; carriers in this subgroup spend on average more than 50 percent of their total cost on expenditures to the independent trucker (O2).

The four-cluster solution is characterized by the clustering of labor intensive firms: cluster 4.2 groups carriers with a significantly large labor in traffic cost component (O4), while cluster 4.3 groups those having a large labor in administration cost component (A4). Both groups were formed from cluster 2.2. The mean profile of cluster 4.4 is similar to that of its parent cluster.

The seven-cluster solution is derived from splittings in clusters 4.1 and 4.4. Subgroups 7.1 and 7.2, derived from cluster 4.1, show the independent trucker to be the major component in their total cost, but in quite different proportions. The three subgroups formed from cluster 4.4 (clusters 7.5, 7.6 and 7.7) are distinguished by large shares of maintenance and repair (O1), vehicle capital costs (K1), and fuel (O3), respectively. It is interesting to note that, at this level, each group has its mean share profile elevated at one of the six cost components that represent more than 90 percent of the total cost. Moreover, fuel (O3) is always the second, if not the first, major cost component.

Table 4.1: Average Shares by Cluster Structures

cluster	firms	O1	O2	O3	O4	O5	A1	A2	A3	A4	A5	K1	K2	K3
1	1172	20.34	7.17	29.19	13.5	1.71	0.22	1.43	1.33	10.29	1.30	12.43	0.23	0.85
2.1	130	10.66	53.12	14.43	6.06	1.14	0.23	1.43	0.46	4.48	1.19	5.86	0.24	0.70
2.2	1042	21.55	1.44	31.03	14.43	1.79	0.21	1.43	1.43	11.02	1.31	13.25	0.23	0.87
4.1	130	10.66	53.12	14.43	6.06	1.14	0.23	1.43	0.46	4.48	1.19	5.86	0.24	0.70
4.2	24	12.44	0.00	12.87	51.26	1.72	0.08	0.29	1.81	6.17	3.28	7.83	0.38	1.88
4.3	87	13.65	1.24	17.05	17.13	1.09	0.20	0.71	2.36	33.00	1.61	9.94	0.26	1.78
4.4	931	22.52	1.49	32.81	13.23	1.85	0.22	1.53	1.34	9.09	1.23	13.70	0.23	0.76
7.1	63	6.33	69.44	8.86	3.76	0.94	0.13	0.90	0.36	4.01	1.05	3.49	0.18	0.55
7.2	67	14.73	37.78	19.67	8.22	1.33	0.33	1.92	0.57	4.92	1.32	8.08	0.29	0.85
7.3	24	12.44	0.00	12.87	51.26	1.72	0.08	0.29	1.81	6.17	3.28	7.83	0.38	1.88
7.4	87	13.65	1.24	17.05	17.13	1.09	0.20	0.71	2.36	33.00	1.61	9.94	0.26	1.78
7.5	21	50.11	0.00	18.13	9.10	2.61	0.06	1.62	1.26	6.31	0.99	9.32	0.07	0.42
7.6	58	16.84	1.15	22.56	8.81	1.16	0.28	1.28	1.03	8.43	1.22	36.19	0.42	0.63
7.7	852	22.23	1.55	33.87	13.63	1.88	0.22	1.55	1.36	9.20	1.24	12.28	0.22	0.77
9.1	63	6.33	69.44	8.86	3.76	0.94	0.13	0.90	0.36	4.01	1.05	3.49	0.18	0.55
9.2	67	14.73	37.78	19.67	8.22	1.33	0.33	1.92	0.57	4.92	1.32	8.08	0.29	0.85
9.3	24	12.44	0.00	12.87	51.26	1.72	0.08	0.29	1.81	6.17	3.28	7.83	0.38	1.88
9.4	87	13.65	1.24	17.05	17.13	1.09	0.20	0.71	2.36	33.00	1.61	9.94	0.26	1.78
9.5	21	50.11	0.00	18.13	9.10	2.61	0.06	1.62	1.26	6.31	0.99	9.32	0.07	0.42
9.6	58	16.84	1.15	22.56	8.81	1.16	0.28	1.28	1.03	8.43	1.22	36.19	0.42	0.63
9.7	5	14.41	5.55	22.02	9.96	17.73	1.20	0.06	3.11	5.06	10.23	8.49	0.50	1.68
9.8	103	12.96	0.42	23.16	29.40	2.10	0.34	0.76	1.79	9.95	1.38	16.09	0.24	1.41
9.9	744	23.56	1.68	35.43	11.47	1.74	0.20	1.66	1.29	9.13	1.16	11.77	0.21	0.68

Table 4.1: (continued)

cluster	firms	O1	O2	O3	O4	O5	A1	A2	A3	A4	A5	K1	K2	K3
12.1	63	6.33	69.44	8.86	3.76	0.94	0.13	0.90	0.36	4.01	1.05	3.49	0.18	0.55
12.2	67	14.73	37.78	19.67	8.22	1.33	0.33	1.92	0.57	4.92	1.32	8.08	0.29	0.85
12.3	24	12.44	0.00	12.87	51.26	1.72	0.08	0.29	1.81	6.17	3.28	7.83	0.38	1.88
12.4	4	17.71	17.20	10.71	5.96	7.47	0.10	0.44	2.56	26.95	4.42	5.58	0.25	0.64
12.5	83	13.45	0.47	17.36	17.67	0.78	0.20	0.72	2.35	33.29	1.48	10.15	0.26	1.83
12.6	21	50.11	0.00	18.13	9.10	2.61	0.06	1.62	1.26	6.31	0.99	9.32	0.07	0.42
12.7	13	30.14	0.54	10.41	12.28	1.33	0.11	0.32	0.76	11.63	0.81	30.45	0.28	0.94
12.8	45	12.99	1.33	26.08	7.81	1.11	0.33	1.56	1.11	7.50	1.34	37.84	0.46	0.54
12.9	5	14.41	5.55	22.02	9.96	17.73	1.20	0.06	3.11	5.06	10.23	8.49	0.50	1.68
12.10	103	12.96	0.42	23.16	29.40	2.10	0.34	0.76	1.79	9.95	1.38	16.09	0.24	1.41
12.11	115	12.60	0.42	49.70	10.36	1.09	0.11	0.77	1.25	11.09	0.76	10.98	0.20	0.68
12.12	629	25.56	1.92	32.82	11.68	1.86	0.21	1.83	1.30	8.77	1.24	11.92	0.22	0.68

Note: Clusters are numbered from the left to the right according to the tree in Figure 4.1. For instance, cluster 7.6 is the sixth cluster counting from the leftmost one at the seven cluster level.

As the number of partitions rises, the formation of small clusters becomes increasingly evident. Also, the different interactions among production factors begin to appear; the mean share profile of firms becomes distinguished by high shares in more than one input factor. For instance, in the nine-cluster solution, cluster 9.7 is formed by five firms with a particularly high share of "other expenditures" (O5 and A5), while cluster 9.8 has a distinctive profile with respect to labor in traffic (O4), fuel (O3), and vehicle costs (K1). In the twelve-partition structure, clusters 12.7 and 12.8, derived from the capital intensive group 9.6, have distinct behavior regarding expenditures for maintenance and repair (O1) and fuel (O3).

Application of the centroid procedure yields a similar result. The pseudo statistics and CCC (Figure 4.4) strongly indicate the same two partitions in the data, with an almost perfect correspondence with the two average linkage clusters. Less significant solutions of four, seven, and nine clusters are also suggested.

The mean share profiles for the centroid solutions are shown in Table 4.2, in which cluster labeling is matched with that of average linkage. The two-, four-, and seven-cluster solutions are, for all practical purposes, parallel to the subgroups found by the average linkage analysis. Although they have very similar mean share profiles, group sizes are different. This is to be expected since clusters are joined under distinct criteria.[15]

In summary, both algorithms differentiate at least two clusters, with the potential for an even larger number. Although the four- and seven-cluster structures from both analyses differ somewhat in terms of group composition, they are very similar with respect to their share profile. This is not the case for the nine- and twelve-cluster solutions for which profiles and group composition both differ considerably.[16]

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[15] The distance among two clusters is defined, for the average linkage method, as the average distance between pairs of observations, one in each cluster; in the centroid method, the distance is taken as the squared Euclidean distance between their means.

[16] Also, the formation of small clusters with less than 10 firms is more an indication of spurious behavior with respect to cost allocation than of a valid grouping.

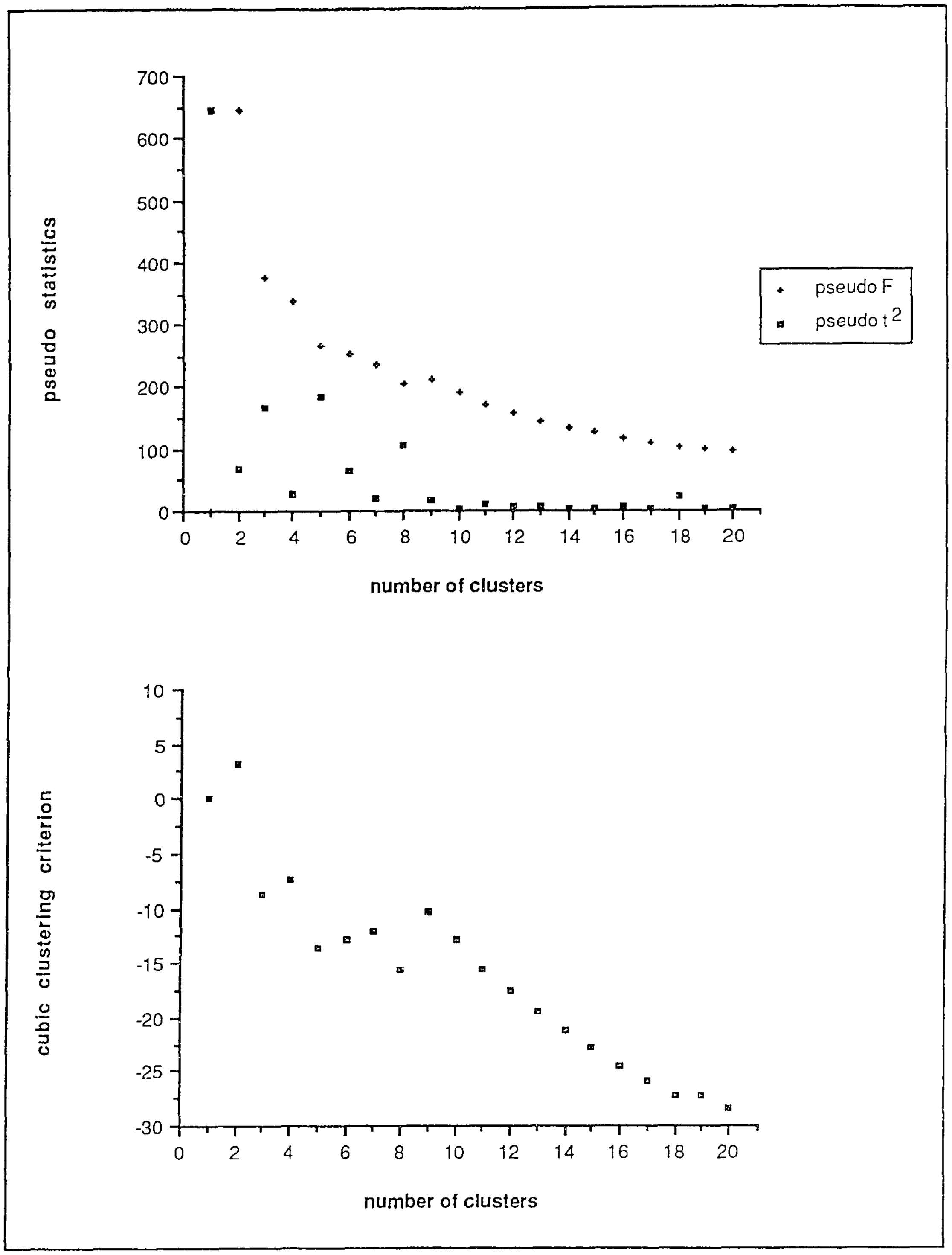

Figure 4.4: Cluster statistics from the centroid algorithm

Table 4.2: Mean Share Profiles from Centroid Clustering

cluster	firms	O1	O2	O3	O4	O5	A1	A2	A3	A4	A5	K1	K2	K3
1	1172	20.34	7.17	29.19	13.50	1.71	0.22	1.43	1.33	10.29	1.30	12.43	0.23	0.85
2.1	134	11.00	52.25	14.89	6.12	1.13	0.23	1.42	0.50	4.48	1.20	5.84	0.24	0.69
2.2	1038	21.54	1.35	31.04	14.46	1.79	0.21	1.43	1.43	11.04	1.31	13.28	0.23	0.87
4.1	134	11.00	52.25	14.89	6.12	1.13	0.23	1.42	0.50	4.48	1.20	5.84	0.24	0.69
4.2	59	11.29	0.79	15.50	43.87	1.42	0.11	0.37	1.65	11.99	2.35	9.15	0.27	1.23
4.3	24	11.37	2.93	14.04	11.95	1.69	0.18	0.50	2.41	43.35	1.59	7.58	0.22	2.19
4.4	955	22.43	1.35	32.42	12.70	1.81	0.22	1.52	1.40	10.17	1.24	13.68	0.23	0.81
7.1	62	6.26	69.75	8.72	3.81	0.95	0.13	0.92	0.36	4.02	1.03	3.33	0.18	0.52
7.2	72	15.08	37.19	20.21	8.11	1.29	0.31	1.86	0.62	4.88	1.34	8.01	0.29	0.83
7.3	59	11.29	0.79	15.50	43.87	1.42	0.11	0.37	1.65	11.99	2.35	9.15	0.27	1.23
7.4	24	11.37	2.93	14.04	11.95	1.69	0.18	0.50	2.41	43.35	1.59	7.58	0.22	2.19
7.5	10	53.63	0.00	16.99	8.62	2.67	0.12	0.91	1.48	7.47	1.03	6.44	0.07	0.59
7.6	27	18.16	2.44	17.33	8.26	1.38	0.54	0.83	1.04	6.36	1.48	41.19	0.48	0.50
7.7	918	22.22	1.33	33.04	12.88	1.82	0.21	1.55	1.41	10.31	1.24	12.95	0.23	0.83
9.1	62	6.26	69.75	8.72	3.81	0.95	0.13	0.92	0.36	4.02	1.03	3.33	0.18	0.52
9.2	72	15.08	37.19	20.21	8.11	1.29	0.31	1.86	0.62	4.88	1.34	8.01	0.29	0.83
9.3	59	11.29	0.79	15.50	43.87	1.42	0.11	0.37	1.65	11.99	2.35	9.15	0.27	1.23
9.4	24	11.37	2.93	14.04	11.95	1.69	0.18	0.50	2.41	43.35	1.59	7.58	0.22	2.19
9.5	10	53.63	0.00	16.99	8.62	2.67	0.12	0.91	1.48	7.47	1.03	6.44	0.07	0.59
9.6	20	12.20	2.94	19.33	7.62	1.04	0.66	0.92	1.07	6.49	1.61	45.03	0.62	0.47
9.7	7	35.20	1.01	11.62	10.11	2.36	0.18	0.59	0.96	5.99	1.09	30.24	0.07	0.58
9.8	56	14.48	0.87	57.20	8.69	1.12	0.14	0.65	1.06	5.09	0.57	9.30	0.15	0.68
9.9	862	22.72	1.36	31.47	13.15	1.86	0.22	1.61	1.43	10.65	1.28	13.19	0.23	0.83

Using the average linkage solution as a reference, Table 4.3 displays the following indices characterizing carriers' operation for each of the cluster structures: total transportation output (*Y*), output per truck (*YPF*), output per carrying capacity (*YPC*), fleet size (*FLT*), total carrying capacity (*CAP*), average truck size (*ACAP*), and cost per unit of output (*CPY*). For comparison purposes, the same indices normalized by the sample mean are shown in Table 4.4.

Although a direct association among share profiles and production indices may not be inferred and generalized (e.g., that a high labor cost share implies that a firm has an average of 5 trucks), a crude analysis of Tables 4.1 and 4.3 reveals that each cluster has a particular characteristic with respect to firm size (fleet), carrying capacity (truck size), and transportation output. It is somewhat surprising, however, that the cost per unit of output remains relatively stable.

In summary, this comparative analysis performed on the basis of average shares indicates the relative importance of each input in each of the groupings. Basically, each structure has an accentuated profile in one production factor. In order to show the influence of the various other inputs on cluster assignment, CART was applied to the two-, four-, and seven-cluster structures suggested by the average linkage method.

4.2.2 CART Results

The binary classification tree generated for the seven average linkage clusters is shown in Figure 4.5. The number of firms in each of the terminal nodes is reported at the bottom of the tree.[17] Table 4.5 summarizes the splits shown in Figure 4.5, and the final classification of the 21 terminal nodes is given in Table 4.6. The accuracy of the classification rule can be assessed from the data in Table 4.7, which displays the *hit-and-miss* matrix. Table 4.8. presents the classification probability matrix.

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[17] The tree was estimated by a 10-fold cross validation using the Gini criterion to assess class separation and unit misclassification costs.

Table 4.3: Motor Carrier Production Profile [a]

cluster	firms	Y *tons*	YPF *tons/veh*	YPC	FLT	CAP *tons*	ACAP *tons/veh*	CPY[b] *US$/ton*
1	1172	17040.6	1998.9	149.6	7.7	117.5	14.0	28.31
2.1	130	77540.3	5499.6	366.0	20.1	345.2	15.9	27.66
2.2	1042	9492.6	1562.2	122.6	6.1	89.1	13.8	26.78
4.1	130	77540.3	5499.6	366.0	20.1	345.2	15.9	27.66
4.2	24	22383.6	998.3	95.3	29.0	200.5	11.1	21.73
4.3	87	4692.2	1151.0	110.4	5.2	47.2	11.2	31.22
4.4	931	9608.9	1615.2	124.4	5.6	90.1	14.1	26.50
7.1	63	90556.1	8378.0	555.9	18.2	312.4	15.4	26.76
7.2	67	65301.6	2793.0	187.5	21.8	376.0	16.2	28.50
7.3	24	22383.6	998.3	95.3	29.0	200.5	11.1	21.73
7.4	87	4692.2	1151.0	110.4	5.2	47.2	11.2	31.22
7.5	21	4703.6	1154.2	95.2	5.3	66.2	13.6	40.88
7.6	58	7954.3	1422.5	89.8	6.1	102.1	15.8	28.05
7.7	852	9842.5	1639.7	127.5	5.6	89.9	14.0	26.04
9.1	63	90556.1	8378.0	555.9	18.2	312.4	15.4	26.76
9.2	67	65301.6	2793.0	187.5	21.8	376.0	16.2	28.50
9.3	24	22383.6	998.3	95.3	29.0	200.5	11.1	21.73
9.4	87	4692.2	1151.0	110.4	5.2	47.2	11.2	31.22
9.5	21	4703.6	1154.2	95.2	5.3	66.2	13.6	40.88
9.6	58	7954.3	1422.5	89.8	6.1	102.1	15.8	28.05
9.7	5	5687.0	2107.6	214.7	4.0	30.8	7.2	73.00
9.8	103	9458.1	1085.3	115.6	6.7	90.7	11.0	24.46
9.9	744	9923.6	1713.2	128.5	5.5	90.2	14.5	25.94
12.1	63	90556.1	8378.0	555.9	18.2	312.4	15.4	26.76
12.2	67	65301.6	2793.0	187.5	21.8	376.0	16.2	28.50
12.3	24	22383.6	998.3	95.3	29.0	200.5	11.1	21.73
12.4	4	10506.8	598.7	68.2	35.3	197.8	10.2	60.04
12.5	83	4412.0	1177.6	112.4	3.7	40.0	11.3	29.83
12.6	21	4703.6	1154.2	95.2	5.3	66.2	13.6	40.88
12.7	13	9376.9	775.5	62.3	7.8	111.0	12.4	28.51
12.8	45	7543.4	1609.4	97.7	5.7	99.5	16.8	27.92
12.9	5	5687.0	2107.6	214.7	4.0	30.8	7.2	73.00
12.10	103	9458.1	1085.3	115.6	6.7	90.7	11.0	24.46
12.11	115	5012.7	1665.5	128.6	3.2	47.7	13.8	24.99
12.12	629	10821.4	1722.0	128.5	5.9	98.0	14.6	26.11

Note: a. see variables definition in the text.
b. Dollar values are based on an average exchange rate of Cr$/US$ 93.18 during 1981.

Table 4.4: Production Profiles Normalized by the Sample Mean

cluster	firms	Y	YPF	YPC	FLT	CAP	ACAP	CPY
1	1172	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.1	130	4.550	2.751	2.447	2.614	2.938	1.129	1.029
2.2	1042	0.557	0.782	0.819	0.799	0.758	0.984	0.996
4.1	130	4.550	2.751	2.447	2.614	2.938	1.129	1.029
4.2	24	1.314	0.499	0.637	3.781	1.707	0.791	0.808
4.3	87	0.275	0.576	0.738	0.673	0.402	0.798	1.161
4.4	931	0.564	0.808	0.832	0.733	0.767	1.006	0.986
7.1	63	5.314	4.191	3.717	2.375	2.658	1.099	0.996
7.2	67	3.832	1.397	1.253	2.839	3.200	1.157	1.060
7.3	24	1.314	0.499	0.637	3.781	1.707	0.791	0.808
7.4	87	0.275	0.576	0.738	0.673	0.402	0.798	1.161
7.5	21	0.276	0.577	0.636	0.688	0.564	0.971	1.521
7.6	58	0.467	0.712	0.600	0.799	0.869	1.128	1.044
7.7	852	0.578	0.820	0.852	0.730	0.765	0.999	0.969
9.1	63	5.314	4.191	3.717	2.375	2.658	1.099	0.996
9.2	67	3.832	1.397	1.253	2.839	3.200	1.157	1.060
9.3	24	1.314	0.499	0.637	3.781	1.707	0.791	0.808
9.4	87	0.275	0.576	0.738	0.673	0.402	0.798	1.161
9.5	21	0.276	0.577	0.636	0.688	0.564	0.971	1.521
9.6	58	0.467	0.712	0.600	0.799	0.869	1.128	1.044
9.7	5	0.334	1.054	1.436	0.521	0.262	0.513	2.716
9.8	103	0.555	0.543	0.773	0.871	0.772	0.781	0.910
9.9	744	0.582	0.857	0.859	0.712	0.768	1.032	0.965
12.1	63	5.314	4.191	3.717	2.375	2.658	1.099	0.996
12.2	67	3.832	1.397	1.253	2.839	3.200	1.157	1.060
12.3	24	1.314	0.499	0.637	3.781	1.707	0.791	0.808
12.4	4	0.617	0.299	0.456	4.590	1.683	0.728	2.234
12.5	83	0.259	0.589	0.751	0.485	0.340	0.801	1.110
12.6	21	0.276	0.577	0.636	0.688	0.564	0.971	1.521
12.7	13	0.550	0.388	0.417	1.012	0.945	0.883	1.061
12.8	45	0.443	0.805	0.653	0.738	0.847	1.198	1.039
12.9	5	0.334	1.054	1.436	0.521	0.262	0.513	2.716
12.1	103	0.555	0.543	0.773	0.871	0.772	0.781	0.910
12.11	115	0.294	0.833	0.860	0.413	0.406	0.981	0.930
12.12	629	0.635	0.861	0.859	0.767	0.834	1.042	0.972

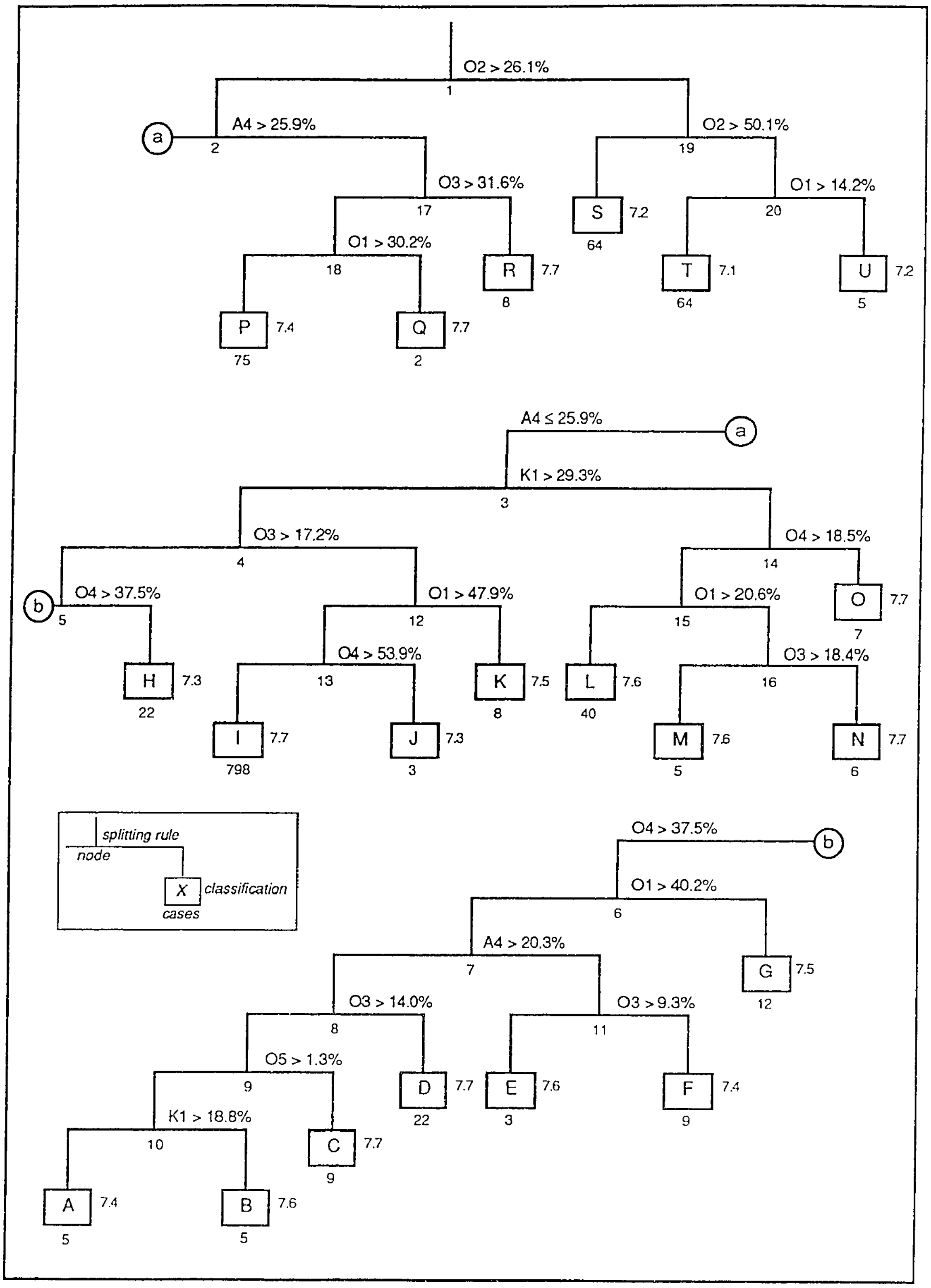

Figure 4.5: CART classification tree for the seven-cluster solution

Table 4.5: CART Splitting

node	splitting variable	split at	next nodes [a]		terminal nodes	
			left	right	left	right
1	O2	26.1	2	19		
2	A4	25.9	3	17		
3	K1	29.3	4	14		
4	O3	17.2	5	12		
5	O4	37.5	6			H
6	O1	40.2	7			G
7	A4	20.3	8	11		
8	O3	14.0	9			D
9	O5	1.3	10			C
10	K1	18.8			A	B
11	O3	9.3			E	F
12	O1	47.9	13			K
13	O4	53.9			I	J
14	O4	18.5	15			O
15	O1	20.6		16	L	
16	O3	18.4			M	N
17	O3	31.6	18			R
18	O1	30.2			P	Q
19	O2	50.1		20	S	
20	O1	14.2			T	U

Note: a. a case goes left if the splitting variable is less or equal to the critical value.

Table 4.6: Terminal Node Information

node	cases	cluster 1	 2	 3	 4	 5	 6	 7	class assignment
A	5	0	1	1	3	0	0	0	4
B	5	0	0	0	0	0	4	1	6
C	9	0	0	0	0	0	1	8	7
D	22	0	0	0	0	0	0	22	7
E	3	0	0	0	0	0	2	1	6
F	9	0	0	0	8	0	0	1	4
G	12	0	0	0	0	12	0	0	**5**
H	22	0	0	22	0	0	0	2	**3**
I	798	0	0	0	4	2	5	787	**7**
J	3	0	0	3	0	0	0	0	3
K	8	0	0	0	0	7	0	1	5
L	40	0	0	0	0	0	40	0	**6**
M	5	0	0	0	0	0	5	0	6
N	6	0	0	0	0	0	0	6	7
O	7	0	0	0	0	0	0	7	7
P	75	0	0	0	72	0	0	3	**4**
Q	2	0	0	0	0	0	0	2	7
R	8	0	0	0	0	0	0	8	7
S	64	0	60	0	0	0	1	3	**2**
T	64	63	1	0	0	0	0	0	**1**
U	5	0	5	0	0	0	0	0	2

Table 4.7: Classification Matrix

class	cluster							total
assignment	1	2	3	4	5	6	7	
1	61	3	0	0	0	0	0	64
2	2	59	0	0	0	0	2	63
3	0	0	20	2	0	0	3	25
4	0	1	1	72	0	0	14	88
5	0	0	0	0	15	0	3	18
6	0	0	0	1	1	46	8	56
7	0	4	3	12	5	12	822	858
total	63	67	24	87	21	58	852	1172

Table 4.8: Classification Probability Matrix

class	cluster						
assignment	1	2	3	4	5	6	7
1	0.97	0.04	0.00	0.00	0.00	0.00	0.00
2	0.03	0.88	0.00	0.00	0.00	0.00	0.00
3	0.00	0.00	0.83	0.02	0.00	0.00	0.00
4	0.00	0.01	0.04	0.83	0.00	0.00	0.02
5	0.00	0.00	0.00	0.00	0.71	0.00	0.00
6	0.00	0.00	0.00	0.01	0.05	0.79	0.01
7	0.00	0.06	0.13	0.14	0.24	0.21	0.96
total	1.00	1.00	1.00	1.00	1.00	1.00	1.00

It is interesting to note the interactions between factor shares in the classification process. As can be seen from Table 4.6, the terminal nodes that typically describe each of the seven classes, 7.1 through 7.7, are nodes *T*, *S*, *H*, *P*, *G*, *L*, and *I*, respectively. Using the splitting rules that determined these nodes' composition, the interactions among factor shares in the clustering of carriers are easily obtained. For example, cluster 7.3 has a typical profile with respect to expenditures of labor in traffic (O4). From Table 4.9, it can

Table 4.9: Factor Share Interactions

cluster	classification rule		
7.1		O2	> 50.1 %
7.2	26.1 % <	O2	≤ 50.1 %
7.3		O2	≤ 26.1 %
		A4	≤ 25.9 %
		K1	≤ 29.3 %
		O3	≤ 17.2 %
		O4	> 37.5 %
7.4		O2	≤ 26.1 %
		A4	> 25.9 %
		O3	≤ 31.6 %
		O1	≤ 30.2 %
7.5		O2	≤ 26.1 %
		A4	≤ 25.9 %
		K1	≤ 29.3 %
		O3	≤ 17.2 %
		O4	≤ 37.5 %
		O1	> 40.2 %
7.6		O2	≤ 26.1 %
		A4	≤ 25.9 %
		K1	> 29.3 %
		O4	≤ 18.5 %
		O1	≤ 40.2 %
7.7		O2	≤ 26.1 %
		A4	≤ 25.9 %
		K1	≤ 29.3 %
		O3	> 17.2 %
		O1	≤ 47.9 %
		O4	≤ 53.9 %

be seen that this group of firms is basically characterized by a low relative usage of hired capacity (O2), labor in administration (A4), and vehicles capital costs (K1), and a very small fuel cost component (O3), which is at most 17.2 percent of the total cost. The cost of labor in traffic represents at least 37.5 percent of the total cost. Although for cluster 7.4 vehicle capital cost (K1) represents on average about 10 percent of the cost, the interaction among O2, A4, O3, and O1 is the determinant of class assignment. The only two groups for which classification is based on one factor share are clusters 7.1 and 7.2. In the absence of more detailed and specific information about an individual firm's operation, conclusions can be drawn regarding its operation based on these results.

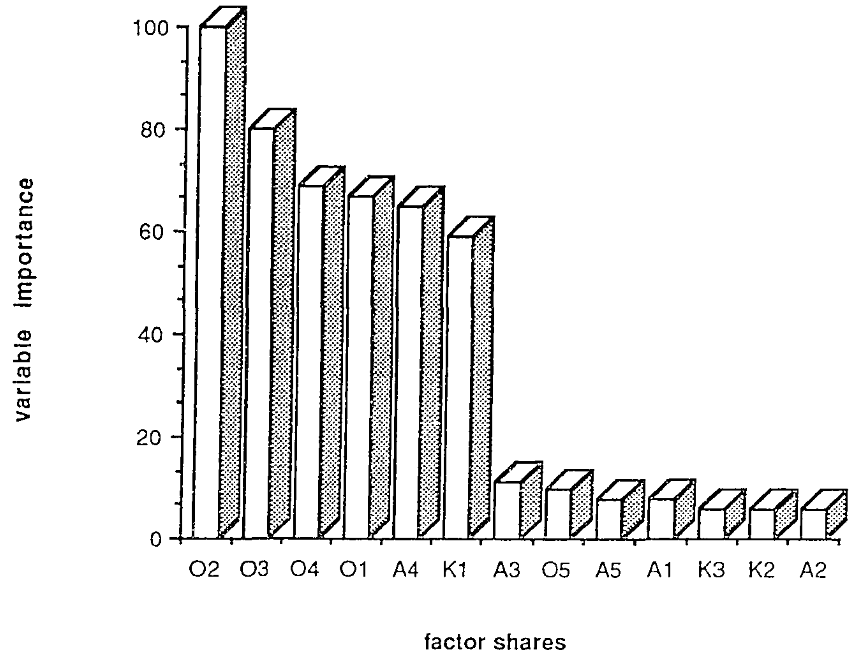

Figure 4.6: **Predictor importance in the classification of the seven groups**

As indicated earlier, one of the issues addressed by the CART algorithm is the relative importance of predictor variables in splitting the data. The measures of importance are shown in Figure 4.6, normalized in such a way that the most important variable has a value of 100. The most relevant predictors are the expenditures in operations (O1, O2, O3, and O4), labor in administration (A4), and vehicle capital costs (K1), which generally account for more than 90 percent of the total cost.

The importance of these six variables is supported by the classification rules determined for the two- and four-cluster solutions depicted in Figure 4.7. It is notable that, while group 4.4 was characterized by a high fuel share, CART translated this in terms of a

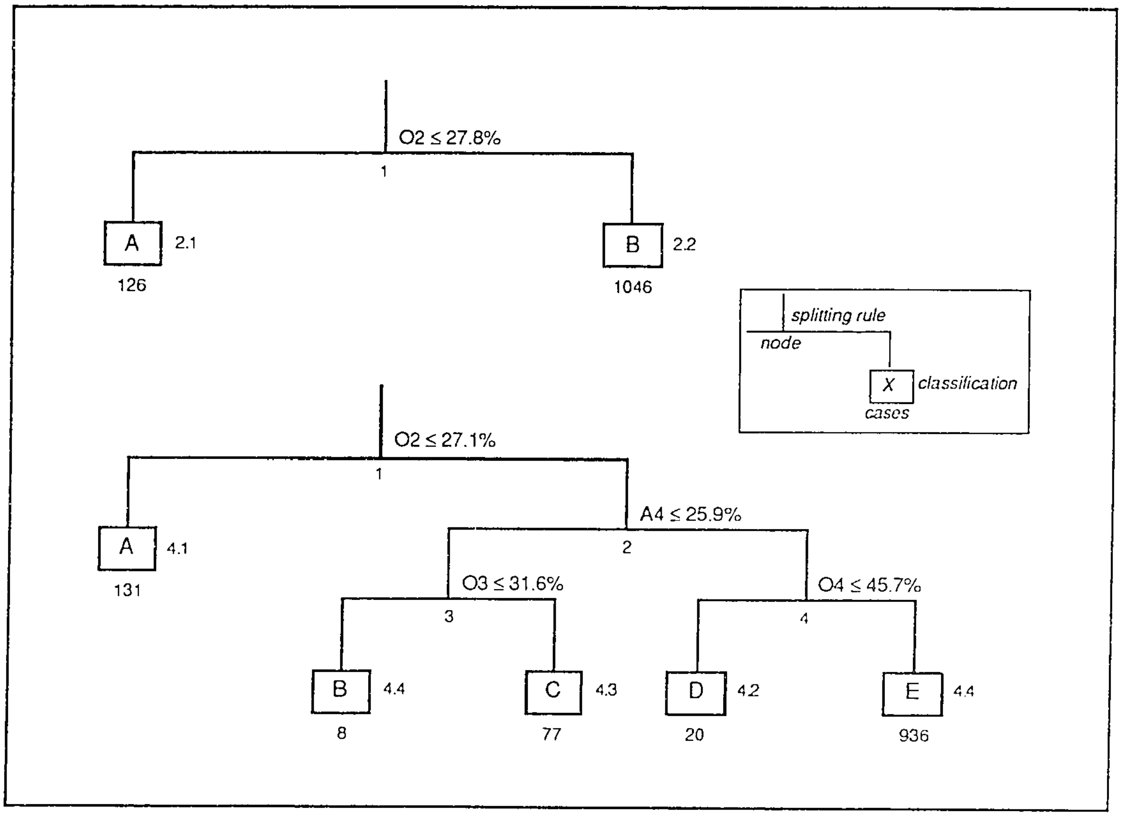

Figure 4.7: CART trees for the case of 2 and 4 clusters

low labor share (A4 ≤ 25.9% and O4 ≤ 45.7%). Cluster 4.3, characterized by a high share of labor in administration, is also described by a fuel share less than 31.6%.

These trees are much simpler than that generated for the seven-cluster solution, and therefore, more easily interpreted. Their simplicity suggests that the solutions with fewer clusters are more a consequence of group behavior with respect to certain variables than a consequence of the interaction among all of them, as in the case of the seven group structure.

4.2.3 Comments

These analyses show that the independent trucker as a factor of production plays a striking role in group discrimination. The two clustering methods applied, average linkage and centroid, suggest two major groups of trucking firms, and as seen from CART trees and mean share profiles, these two groups are distinctly different in their use of the owner-operator. Given the fact that just about 25 percent of the sample (295 out of 1172 firms) reported the use of the owner-operator, the dichotomy *owner-operator vs. no owner-operator* seems to be the determining factor in cluster separation. For this reason a *second-order* analysis was performed; the same methodology was applied to (a) the set of firms using the independent trucker, and (b) to the set of firms not using the independent trucker. The results of these analyses are presented below.

4.3 A SECOND-ORDER ANALYSIS

As in the *first-order* analysis, two clustering algorithms were applied in order to assess the stability of the suggested solutions. Again, both procedures yielded similar results when the number of clusters was small. As the number of clusters increased, however, the correspondence between groups deteriorated substantially. In the interest of consistency, the focus will be on the average linkage analysis.

4.3.1 Firms Purchasing Autonomous Capacity

For the sample of firms using the owner-operator, the average linkage procedure yielded the dendogram depicted in Figure 4.8. Two possible structures, one with two clusters, another with five clusters, are suggested by the pseudo statistics and cubic clustering criterion shown in Figure 4.9.

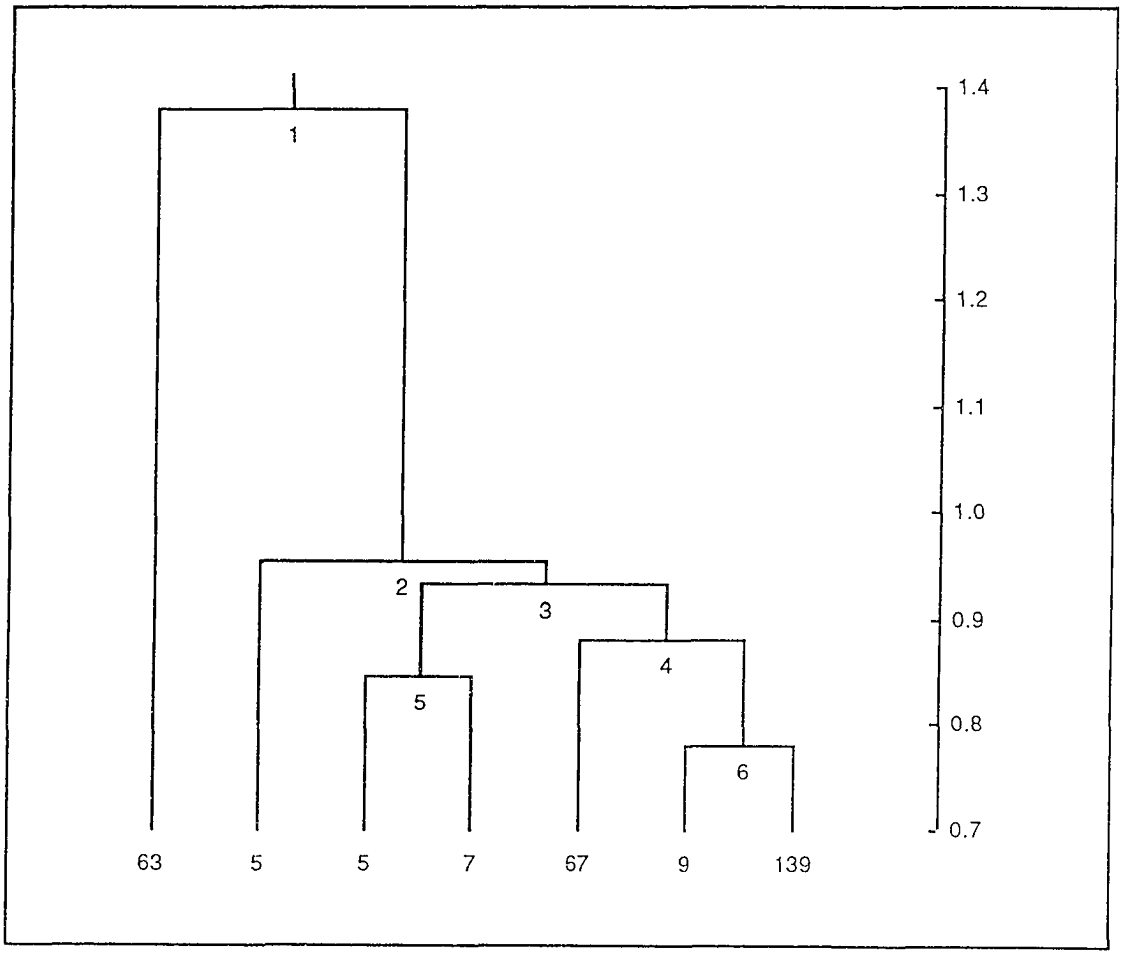

Figure 4.8: Average linkage between clusters of firms using the independent trucker

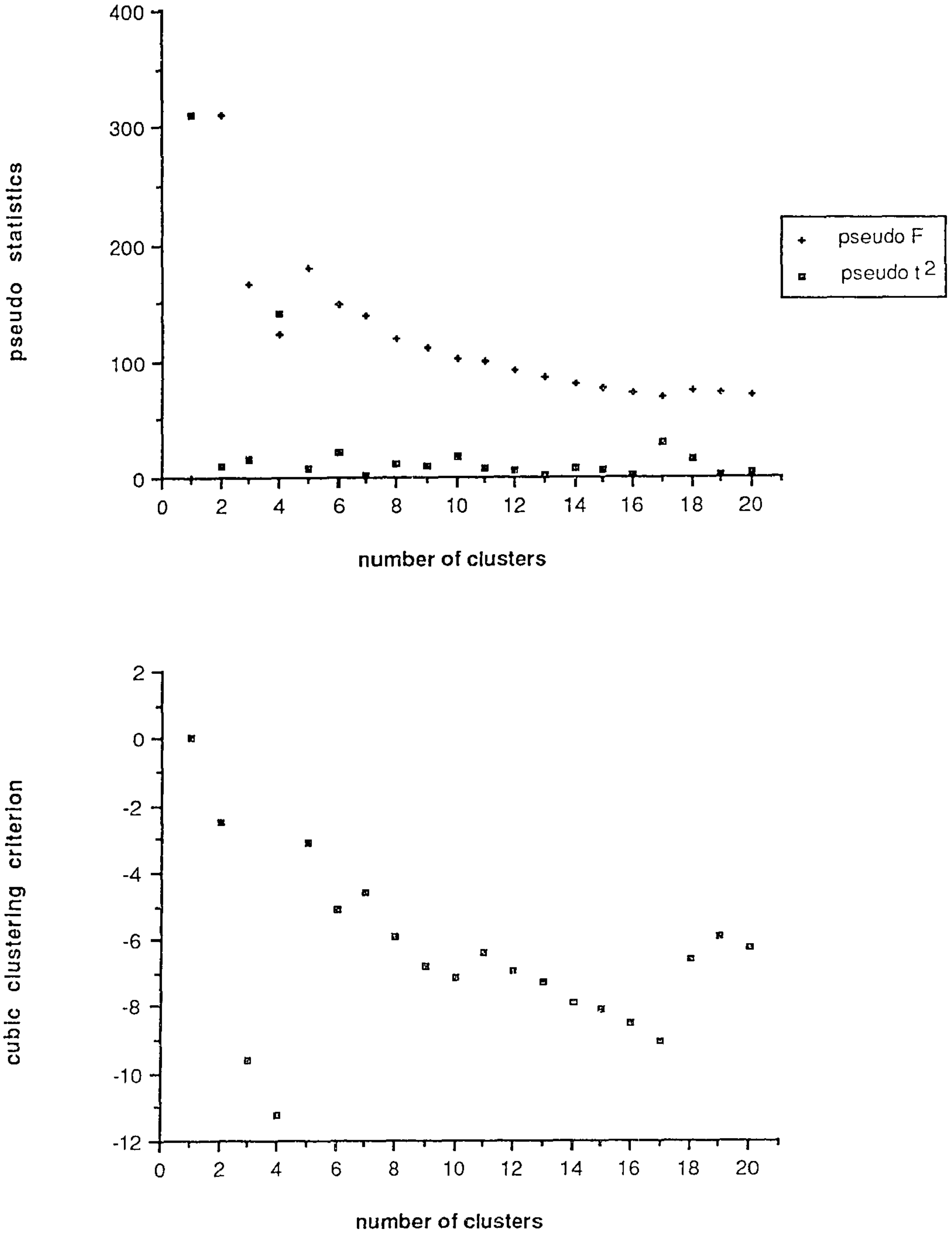

Figure 4.9: Cluster statistics for the sample of firms using hired capacity

Table 4.10 presents the average production characteristics of each cluster. The corresponding average share profiles are shown in Table 4.11. It can be seen that there is a perfect match between clusters 5.1 and 5.4 here, and clusters 7.1 and 7.2 respectively, from the previous analysis (Table 4.1). These two groups of firms are those dependent mostly upon the services of the owner-operator, and have a completely different cost allocation structure from the rest of the sample. It is interesting to note that the two smallest groups, clusters 5.2 and 5.3, have the largest and smallest average truck size (*ACAP*), respectively.

CART trees for both structures are shown in Figure 4.10; class assignment is indicated at the terminal nodes. Once again, it is interesting to observe that the right branch in the five-cluster tree replicates the two-cluster classification tree, indicating the special behavior of clusters 5.1 and 5.4 as compared with the other clusters. Moreover, the only substantial difference between cluster 5.2 and 5.5 seems to be the share of vehicle capital costs (K1).

Table 4.10: Production Profile of Carriers using the Independent Trucker [a]

cluster	firms	Y *tons*	YPF *tons/veh*	YPC	FLT	CAP *tons*	ACAP *tons/veh*	CPY *Cr$/ton*
1	295	50761.2	3592.3	245.3	16.9	280.3	15.4	2968.6
2.1	63	90556.1	8378.0	555.9	18.2	312.4	15.4	2625.9
2.2	232	39954.8	2292.8	160.9	16.6	271.6	15.3	3061.7
5.1	63	90556.1	8378.0	555.9	18.2	312.4	15.4	2625.9
5.2	5	21352.8	1437.5	77.7	18.6	300.8	17.3	2827.3
5.3	12	13447.0	1093.1	136.6	21.4	149.8	10.0	4656.0
5.4	67	65301.6	2793.0	187.5	21.8	376.0	16.3	2796.4
5.5	148	31257.9	2192.5	153.6	13.7	233.2	15.3	3060.4

Note: a. see variables definition in Section 4.2.1.

Table 4.11: Mean Share Profiles

cluster	firms	O1	O2	O3	O4	O5	A1	A2	A3	A4	A5	K1	K2	K3
1	295	16.37	28.49	22.92	9.32	1.90	0.34	1.98	0.67	6.39	1.62	8.70	0.32	0.99
2.1	63	6.33	69.44	8.86	3.76	0.94	0.13	0.90	0.36	4.01	1.05	3.49	0.18	0.55
2.1	232	19.09	17.37	26.74	10.83	2.16	0.39	2.27	0.76	7.03	1.77	10.12	0.35	1.11
5.1	63	6.33	69.44	8.86	3.76	0.94	0.13	0.90	0.36	4.01	1.05	3.49	0.18	0.55
5.2	5	9.27	11.97	17.36	7.22	1.61	0.06	2.47	0.59	6.66	1.45	40.00	0.52	0.81
5.3	12	12.47	9.83	11.47	21.45	3.38	0.21	0.67	1.59	22.54	3.16	9.88	0.19	3.17
5.4	67	14.73	37.78	19.67	8.22	1.33	0.33	1.92	0.57	4.92	1.32	8.08	0.29	0.85
5.5	148	21.93	8.93	31.50	11.27	2.46	0.45	2.55	0.78	6.74	1.87	10.05	0.39	1.07

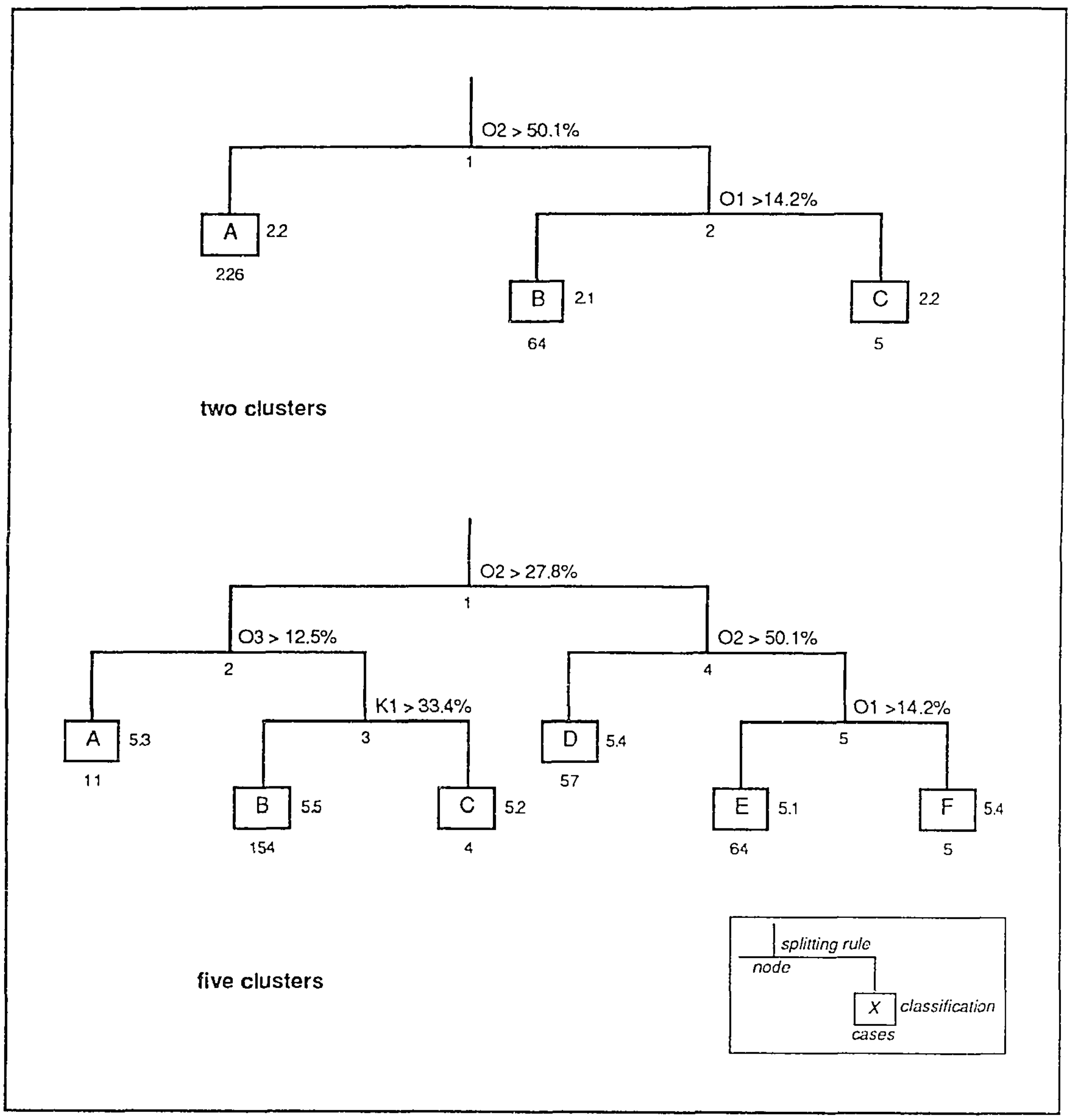

Figure 4.10: Classification trees for the two structures determined for carriers using the independent trucker

The classification power of the variables is displayed in Figure 4.11. As in the seven-cluster analysis, only a few variables have discriminatory power. In this case, labor shares (O4 and A4) have a lower rank. The independent trucker is still the most relevant classifier, which is somewhat surprising. One reason for this is certainly the varying levels of participation of O2 in the total cost among clustered firms.

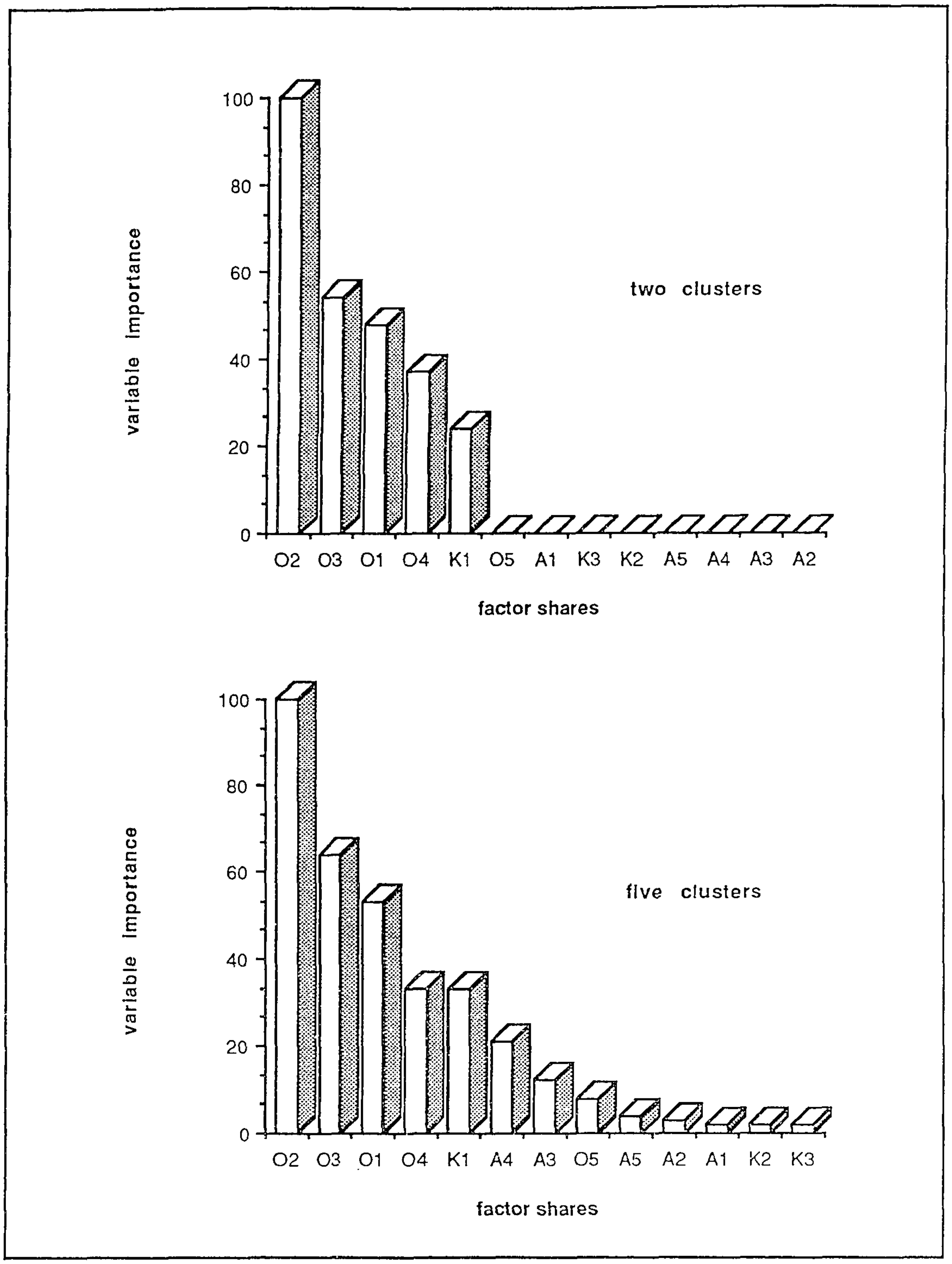

Figure 4.11: Predictor ranking in the classification of the two structures determined for carriers with the independent trucker

4.3.2 Firms Not Purchasing Autonomous Capacity

Figure 4.12 displays the average linkage for the last six clusters joined. Two significant solutions at the three and six cluster levels are indicated by the statistics shown in Figure 4.13. In contrast to the sample involving the independent trucker, the pseudo statistics and clustering criterion for this sample have much more variability. This suggests a grainy distribution of firms, i.e., that in fact the sample has a large number of small groups of motor carriers that are similar to each other.

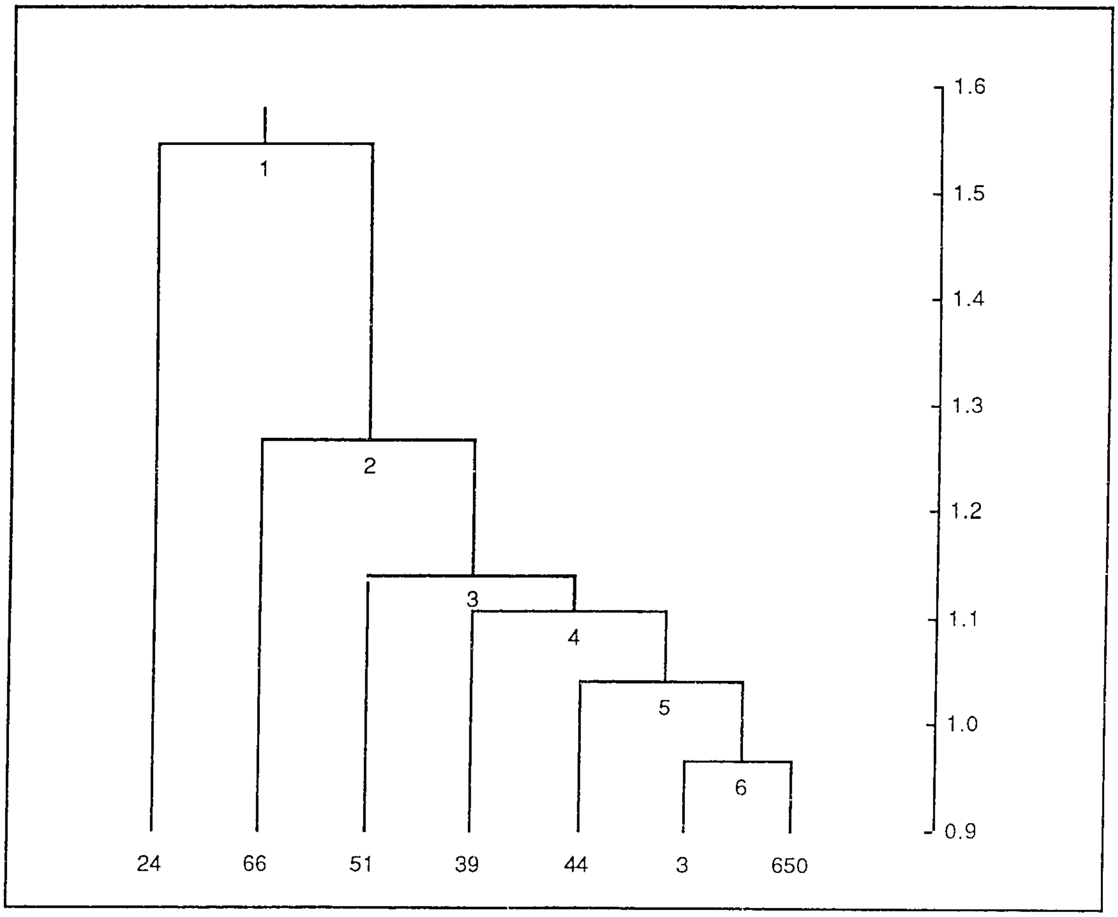

Figure 4.12: Clustering carriers without the independent trucker

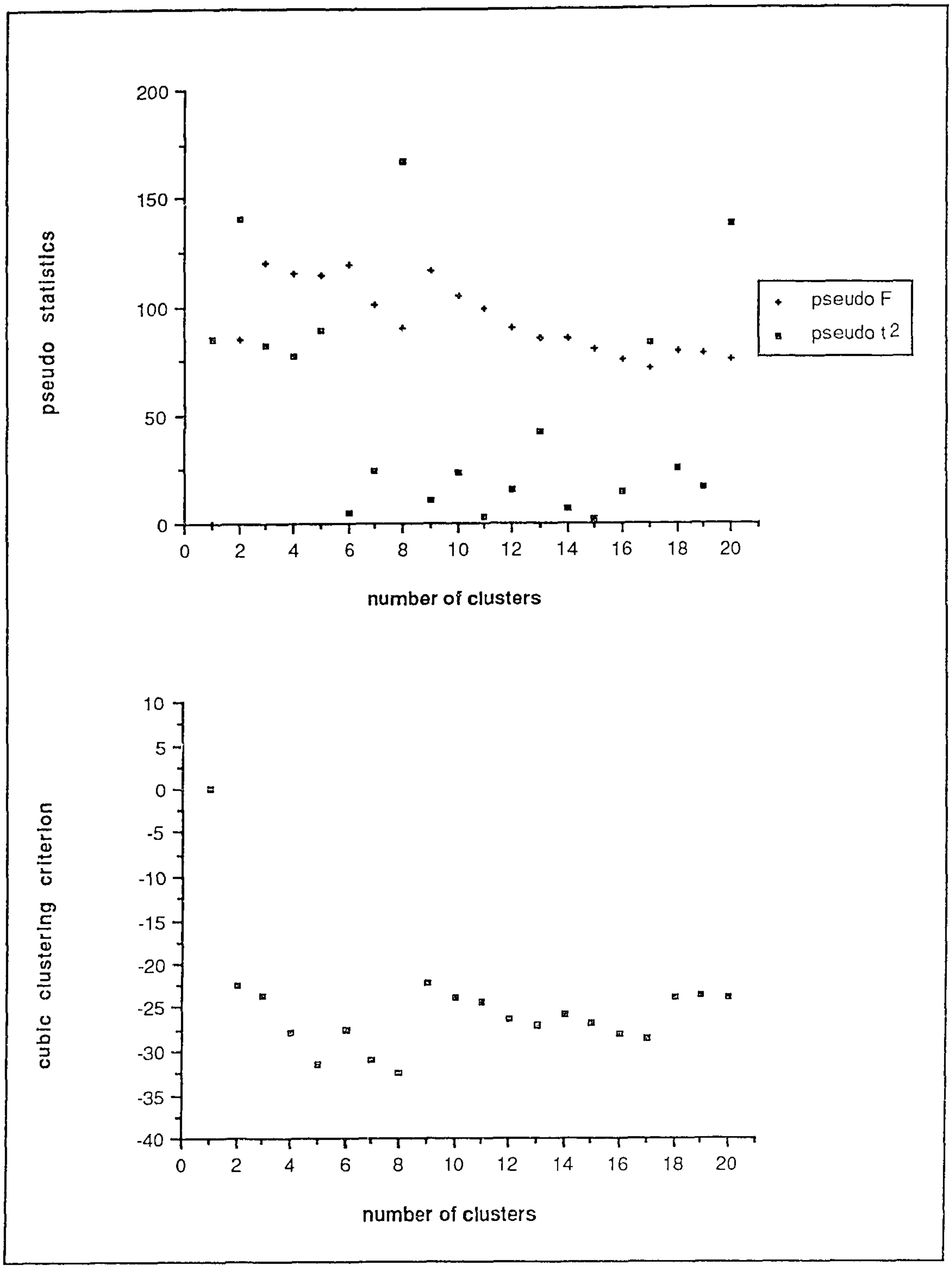

Figure 4.13: Cluster statistics for the sample not using hired capacity

This analysis confirms the earlier findings that firms using hired capacity tend to be more specialized in terms of transportation services. Moreover, since firms using the independent trucker tend to be larger, with on average, four times the number of trucks, size and specialization could explain the smaller number of heterogeneous groups found.

For comparison purposes the average production attributes and share profiles of each group are shown in Table 4.12 and 4.13, respectively. Cluster 6.1 (3.1) is the same as cluster 7.3 of the first order analysis. There is a strong correspondence between clusters 6.2 and 7.4, 6.3 and 7.5, 6.4 and 7.6, and 6.6 with 7.7. Cluster 6.4, with the smallest fleet size and the highest fuel share, is the only one with no close counterpart in the results from the whole-sample analysis.[18]

Table 4.12: Production Profile of Firms without the Independent Trucker [a]

cluster	firms	Y *tons*	YPF *tons/veh*	YPC	FLT	CAP *tons*	ACAP *tons/veh*	CPY *Cr$/ton*
1	877	5697.9	1463.0	117.4	4.6	62.7	13.6	2526.3
3.1	24	22383.6	998.3	95.3	29.0	200.5	11.1	2132.5
3.2	66	4449.4	1236.1	118.8	3.1	32.1	11.3	2841.8
3.3	787	5293.7	1496.2	117.9	3.9	61.1	13.9	2511.8
6.1	24	22383.6	998.3	95.3	29.0	200.5	11.1	2132.5
6.2	66	4449.4	1236.1	118.8	3.1	32.1	11.3	2841.8
6.3	51	5132.2	1420.5	90.3	4.1	71.8	15.8	2796.8
6.4	39	4953.2	1521.0	123.7	4.6	57.6	14.2	3345.4
6.5	44	5858.9	2117.8	152.8	2.9	39.3	14.4	2445.2
6.6	653	5288.6	1458.7	117.4	4.0	62.0	13.7	2444.2

Note: a. see variables definition in Section 4.2.1.

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[18] See Tables 4.1 and 4.3.

Table 4.13: Mean Share Profiles

cluster	firms	O1	O2	O3	O4	O5	A1	A2	A3	A4	A5	K1	K2	K3
1	877	21.68	0.00	31.30	14.91	1.65	0.18	1.25	1.55	11.60	1.19	13.68	0.21	0.80
3.1	24	12.44	0.00	12.87	51.26	1.72	0.08	0.29	1.81	6.17	3.28	7.83	0.38	1.88
3.2	66	12.58	0.00	16.15	19.27	0.73	0.20	0.77	2.42	34.67	1.40	10.18	0.28	1.36
3.3	787	22.72	0.00	33.13	13.44	1.73	0.18	1.32	1.47	9.84	1.11	14.16	0.20	0.72
6.1	24	12.44	0.00	12.87	51.26	1.72	0.08	0.29	1.81	6.17	3.28	7.83	0.38	1.88
6.2	66	12.58	0.00	16.15	19.27	0.73	0.20	0.77	2.42	34.67	1.40	10.18	0.28	1.36
6.3	51	16.95	0.00	23.43	8.85	1.10	0.30	1.13	1.06	8.82	1.18	36.13	0.42	0.62
6.4	39	46.15	0.00	22.01	8.53	1.88	0.24	3.67	1.13	6.48	0.89	8.63	0.06	0.33
6.5	44	15.60	0.00	58.05	8.14	1.06	0.06	0.63	1.19	5.32	0.49	8.90	0.08	0.50
6.6	653	22.25	0.00	32.88	14.44	1.81	0.17	1.24	1.54	10.42	1.16	13.12	0.19	0.77

The CART-generated tree for the three-cluster structure is shown in Figure 4.14, while that for the six-cluster is shown in Figure 4.15. Both trees are much more complex than those for the sample with the independent trucker. The interactions among factor shares seem to be stronger at different parts of the data. Predictor ranking is displayed in Figure 4.16; labor in administration (A4), for the three-cluster structure, and fuel (O3) for the six-cluster solution, have the strongest predictive power.

It is noteworthy that labor in administration plays such a strong part. Since it has not appeared in any significant surrogate or competitor split during tree generation, its high rank between predictors is entirely due to the splits shown in the trees, which are directly related to the assignment of firms to group 6.2 (or 3.2). This indicates how the 66 firms in this group are set apart from the other groups with respect to expenditures on labor in administration.

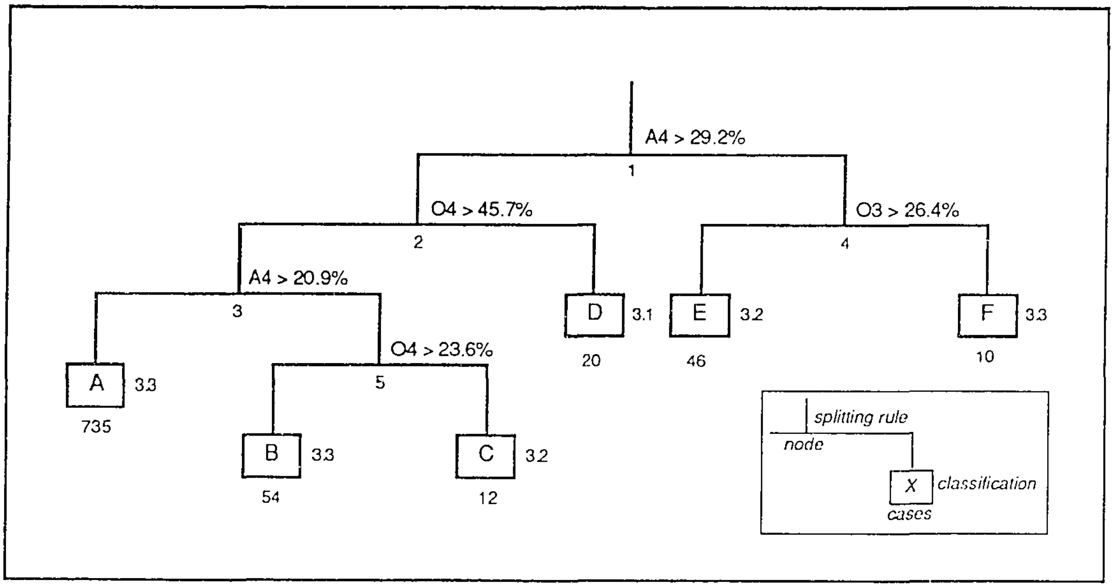

Figure 4.14: Classification tree for the three-cluster solution obtained for carriers not using the independent trucker

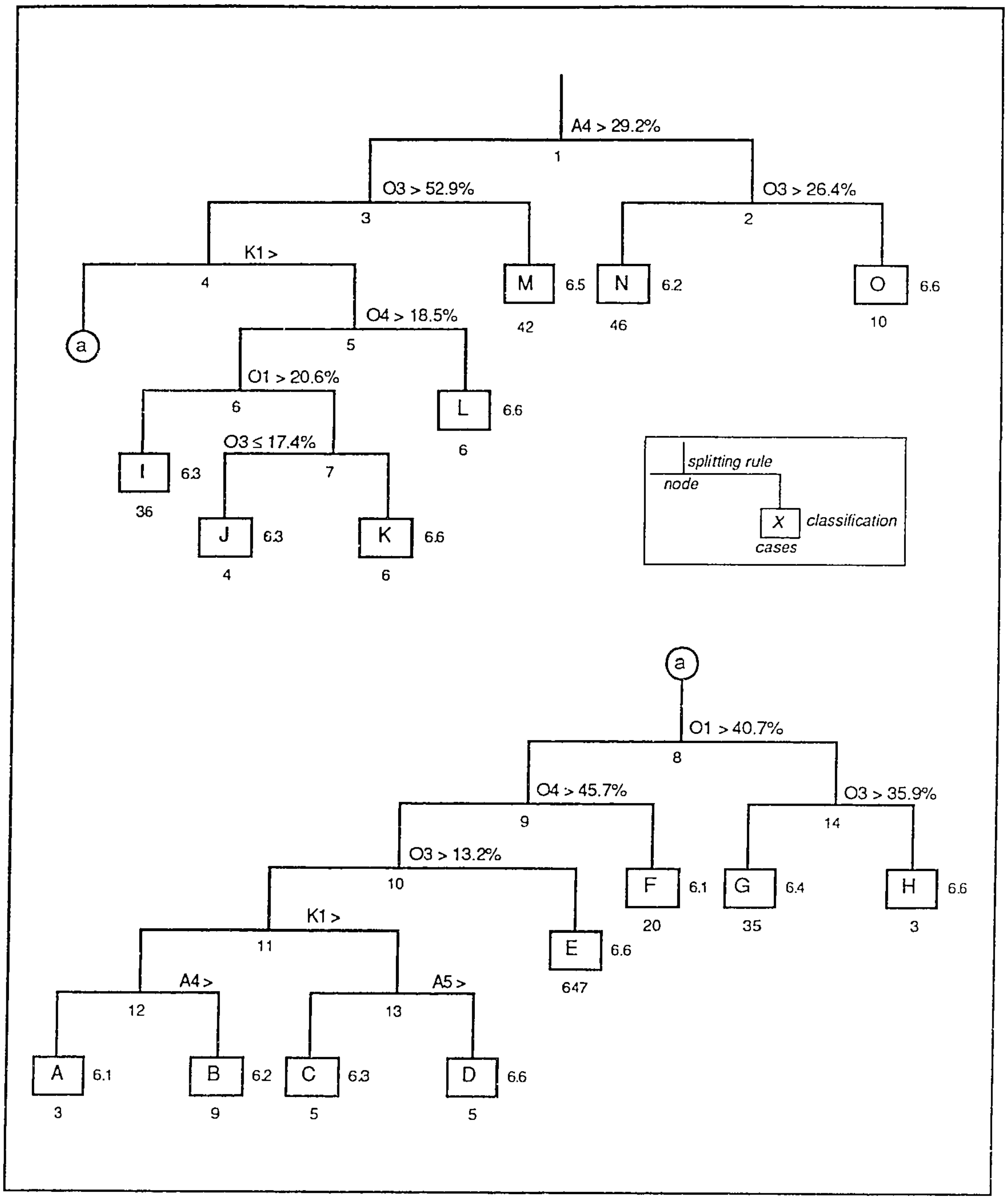

Figure 4.15: Classification tree for the six-cluster solution obtained for carriers not using the independent trucker

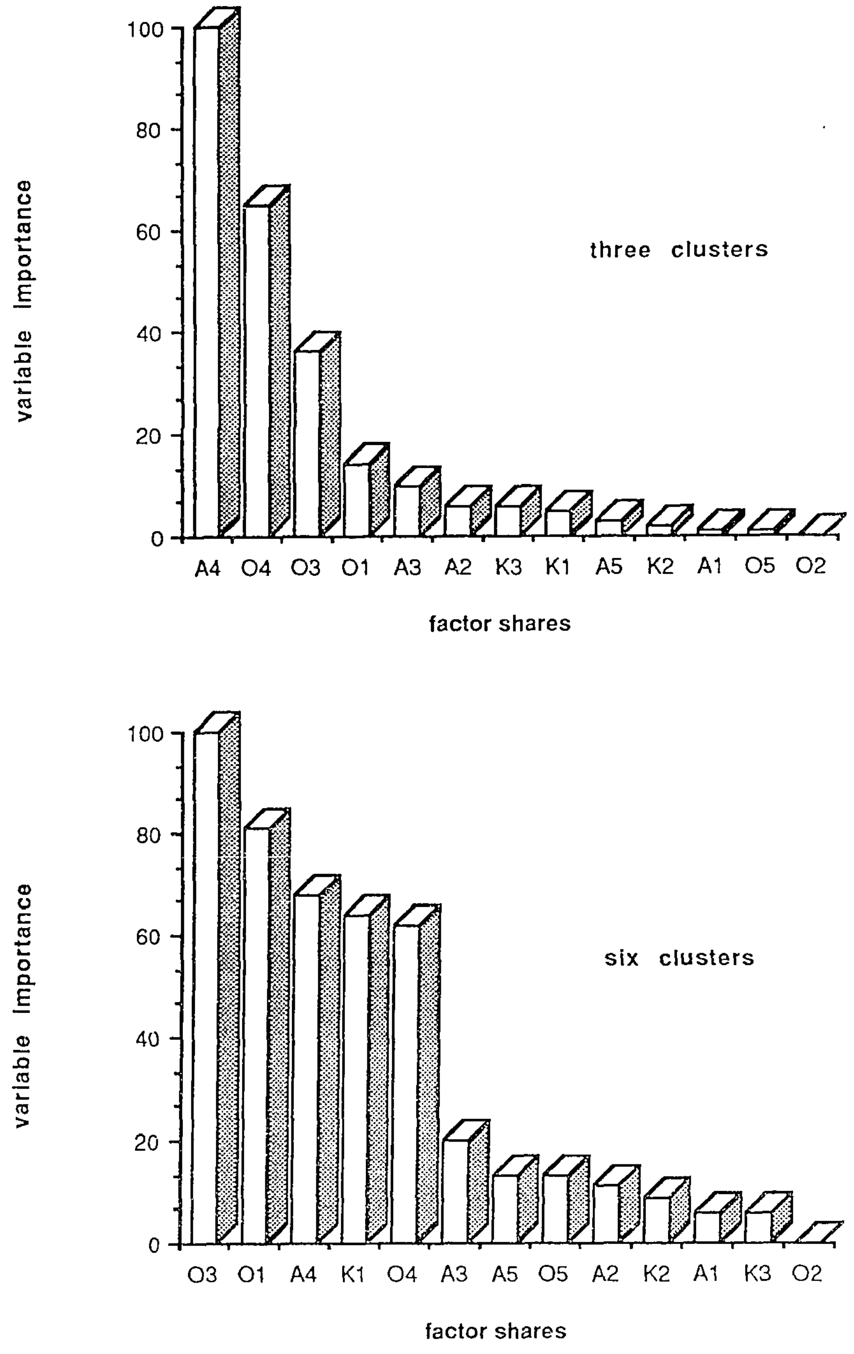

Figure 4.16: Predictor importance in the classification of the two structures determined for carriers not using the independent trucker

Finally, in order to provide an assessment of the relative importance of each cluster in terms of transportation output, Table 4.14 displays their individual contribution to the total output within each subsample. Carriers using the independent trucker, 25 percent of the sample, are responsible for about three quarters of the total production. About 91 percent of output is produced by clusters 5.1, 5.4, 5.5, and 6.6.

Table 4.14: Transportation Output by Cluster (%)

cluster	firms		within subsample	whole sample
2.1	63		38.10	28.57
2.2	232		61.90	46.41
5.1	63		38.10	28.57
5.2	5		0.71	0.53
5.3	12		1.08	0.81
5.4	67		29.22	21.91
5.5	148		30.89	23.16
subsample	295	(25.17%)	100.00	74.98
3.1	24		10.75	2.69
3.2	66		5.88	1.47
3.3	787		83.37	20.86
6.1	24		10.75	2.69
6.2	66		5.88	1.47
6.3	51		5.24	1.31
6.4	39		3.87	0.97
6.5	44		5.16	1.29
6.6	653		69.11	17.29
subsample	877	(74.83%)	100.00	25.02
total	1172		—	100.00

4.4 CONCLUSIONS AND IMPLICATIONS

In summary, the purpose of the analyses presented in this chapter was to identify similar groups of trucking firms based on their distribution of cost shares. The existence of such groups or segments within the industry — the underlying hypothesis of the chapter — is supported by the results: two main groups of carriers were identified on the basis of use of the independent trucker. Within each group, distinct subgroups were also identified, each behaving homogeneously according to a typical cost profile. Moreover, cost share profiles of each of the suggested clusters are closely tied to certain production characteristics like transportation output, fleet size, and truck size, which supports a link between cost allocation and type of service provided by these firms.

The preliminary nature of these results should be emphasized, however. Ideally, it would be useful to obtain comprehensive data on the attributes of trucking operations. These attributes would be used as external validation criteria to evaluate the extent of recovery of the true structure (if one exists) in a given clustering solution. Information on shipment characteristics and market behavior (operating environment), for example, could provide considerable insight into the interpretation of a given structure.

On the basis of these findings the unavoidable question is whether these segments represent different technologies, or to what extent they differ in terms of technical efficiency. As mentioned earlier, this is a crucial point to be investigated since it has major implications with respect to the general representation of the trucking technology. In the next chapter, the hypothesis that these clusters are associated with distinctly different production structures is formally stated and tested.

Chapter V

MODELLING TRUCKING TECHNOLOGY

In this chapter, a set of cost models is set forth to test for the existence of differences in the structure of cost and production of trucking firms grouped according to the analysis developed in the previous chapter. Specifically, the hypothesis that clusters are associated with distinctly different production structures is tested. The underlying strategy is developed in Section 5.1. The empirical cost model is then introduced for each of the two groups of firms under consideration, one formed by the 295 firms making use of the independent trucker (hereafter referred to as group G.295), and another which encompasses the 877 firms that do not make use of the independent trucker (hereafter referred to as group G.877). Section 5.2 discusses the specification and empirical results of the models estimated for each cluster.

5.1 TESTING DIFFERENCES IN COST STRUCTURE

In Chapter IV, it was shown that within each cluster structure, each subgroup behaved homogeneously according to a typical cost share profile. Moreover, it was also shown that the composition of each subgroup could be implicitly determined mainly by the interactions of six factors, and that these factors on average accounted for more than 90 percent of the firm's total cost. Thus, the basic assumption in modeling the structure of technology of the set of trucking firms under analysis is that transportation is produced according to a twice-differentiable production function that relates the flow of output to the service of seven composite inputs: maintenance and repair (O1), purchased transportation (O2), fuel (O3), labor in traffic (O4), labor in administration (A4), capital in transportation means (K1), and all other materials (E1) which aggregates the seven remaining and less relevant factors of the analysis in Chapter IV. With the further assumption that firms are efficient, i.e., that they are cost minimizers, the same technological information provided in the production function can be derived from its dual cost function.

The testing framework introduced in the following section was implemented under the additional assumption that the translog form provides a reasonably satisfactory approximation of the cost function.

5.1.1 Model Specification

The strategy adopted to examine if and how the structure of cost and production differs across clusters may be easily explained with the help of Figure 5.1. Both groups, G.295 and G.877, were similar with respect to the pattern of cluster generation.[1] That is, each successive cluster structure was obtained by the merge of a smaller cluster with another containing the largest number of firms. Figure 5.1 depicts this pattern of clustering. For example, the *m*-cluster structure is obtained from cluster *n.n*-1 joining cluster *n.n* into cluster *m.m*. Similarly, the two-cluster structure is obtained by the union of cluster 3.2 with cluster 3.3 forming cluster 2.2.

The approach proposed to examine the relevance of each cluster partition involves the estimation of a set of cost functions. Each of these functions includes a dummy variable to allow it to shift according to cluster membership. Following the structure in Figure 5.1, each cost model is then estimated for each cluster pair sequentially, starting at the highest

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[1] See Figures 4.8 and 4.12.

level, with two clusters, to the lowest with *n* clusters. That is, using the whole sample first, a cost model is estimated with the binary variable indicating whether the observation came from cluster 2.1 or cluster 2.2. All interaction terms are jointly tested to decide whether or not it is suitable to pool firms from cluster 2.1 with those from cluster 2.2. The same framework is applied to the sample defined by cluster 2.2 to test for the equality between clusters 3.2 and 3.3, and so forth.

For the sample using the independent trucker, G.295, the two- and five-cluster solutions are depicted in Figure 5.2(a). Because clusters 5.2 and 5.3 are too small to provide enough degrees of freedom for the estimation, they were excluded from the analysis and the resulting structure is given in Figure 5.2(b).[2]

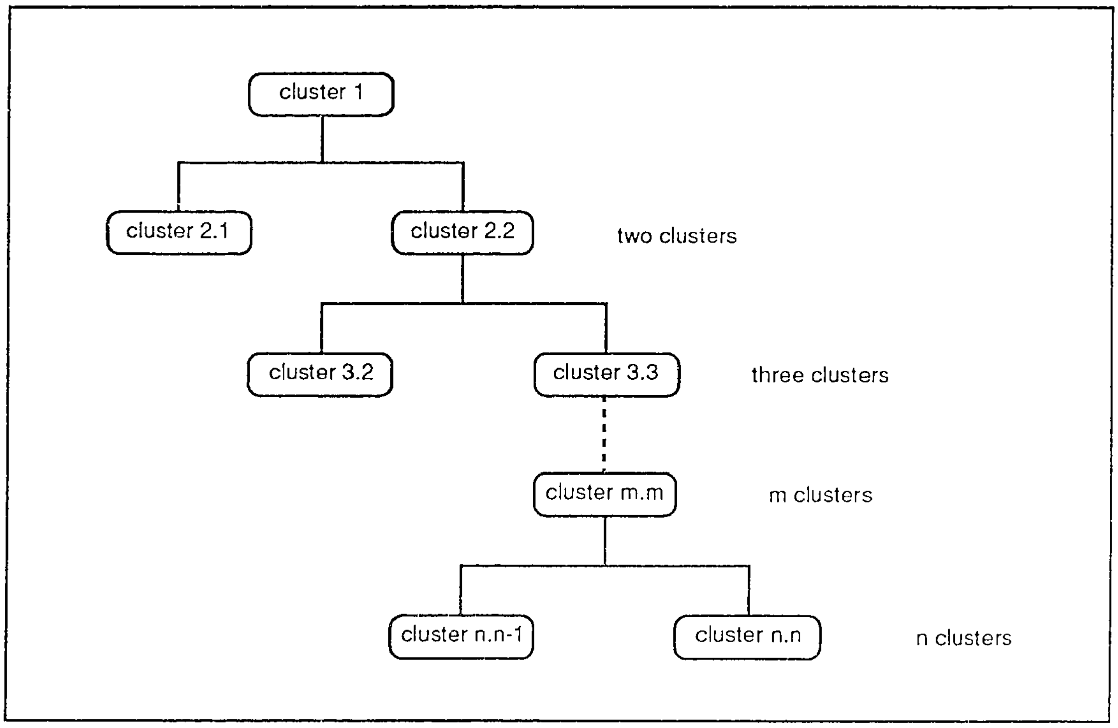

Figure 5.1: Clustering pattern of groups G.295 and G.877

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[2] The notation adopted reflects the size of each group.

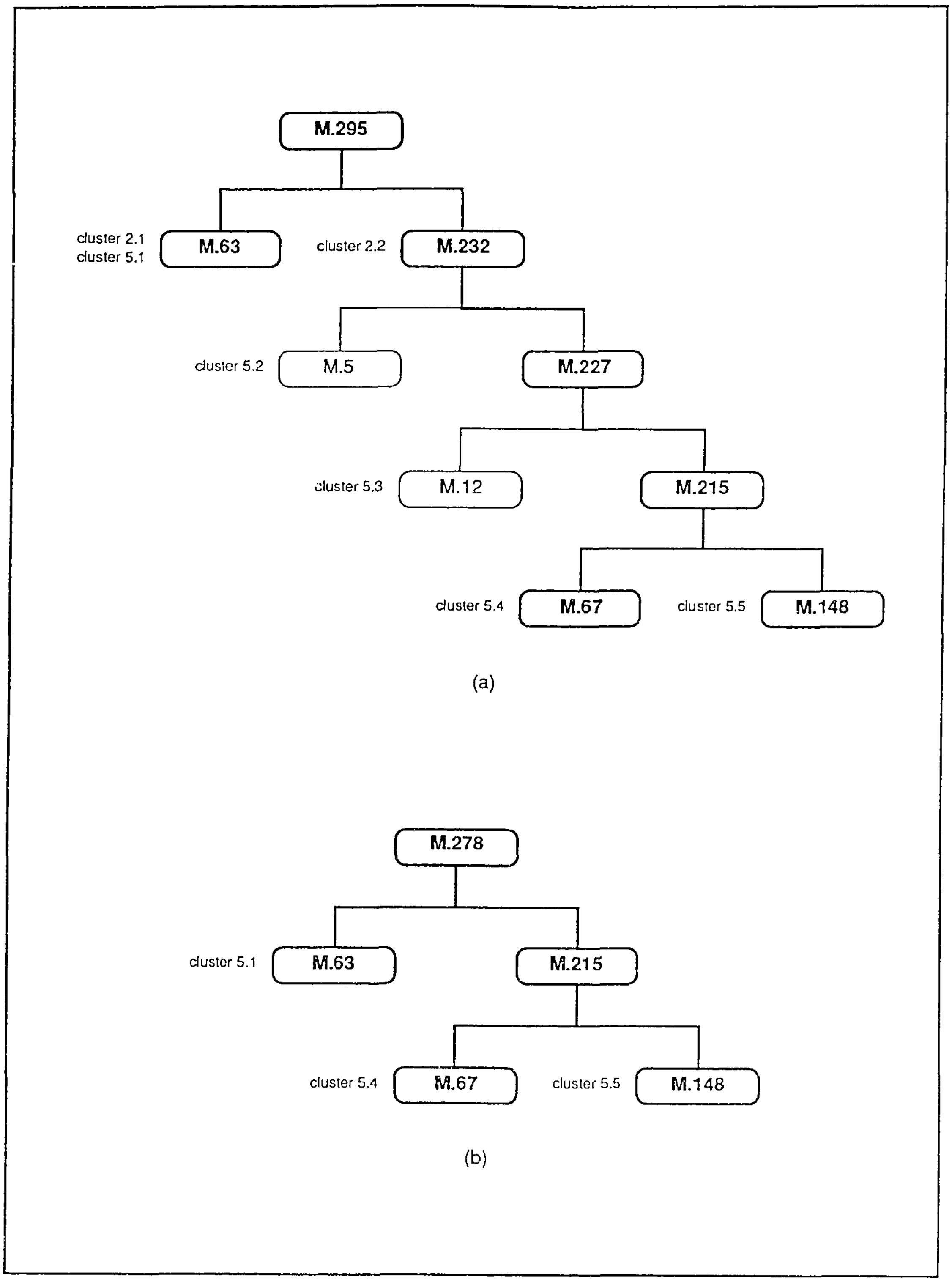

Figure 5.2: Structure of analysis of group G.295

The same problem occurs for the final structure of group G.877, highlighted in Figure 5.3. Again, because of small size, cluster 3.1 (6.1) was excluded from the set of estimation samples.

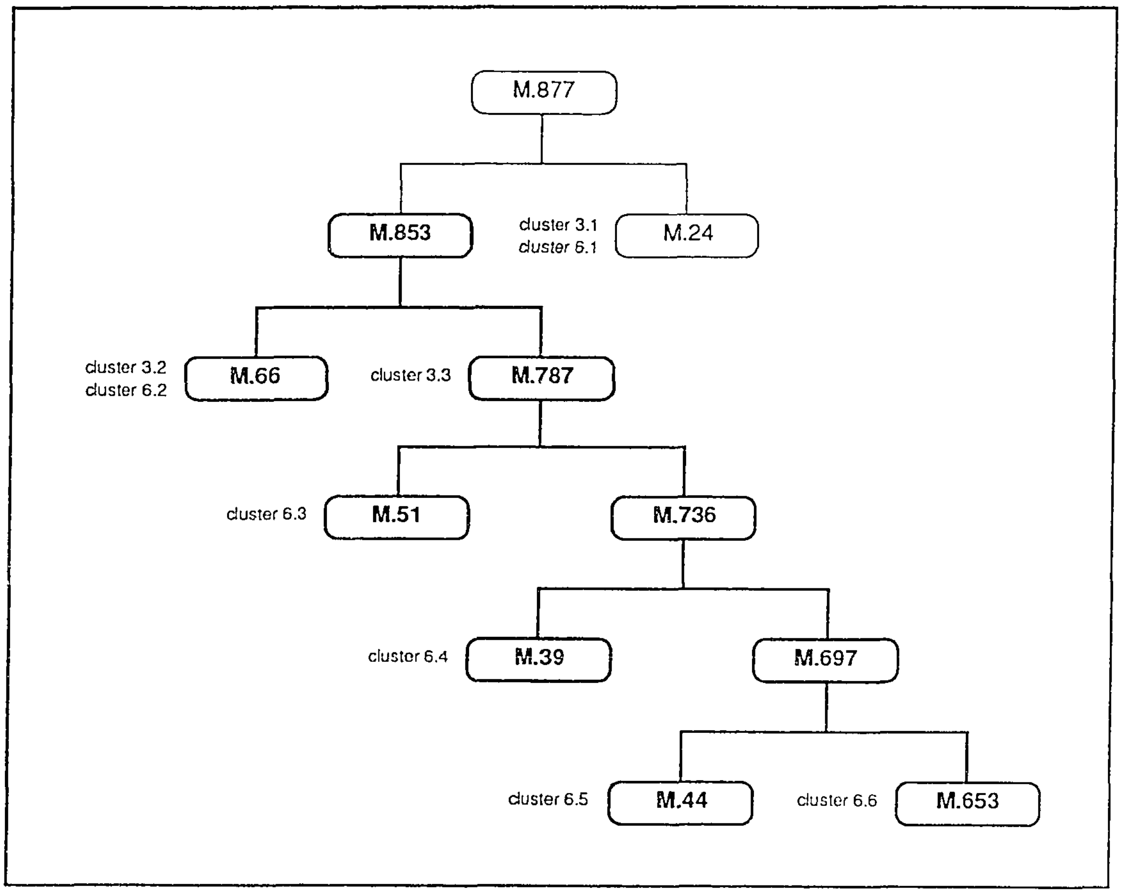

Figure 5.3: Structure of analysis of group G.877

Testing the two structures above involves the specification and estimation of six cost functions, two for group G.295 and four for group G.877, each taking the general form $C = C[y,w,d]$, where y is output, w is the vector of input prices, and d is a dummy variable characterizing cluster membership. These six models are listed in Table 5.1. All models maintain that factor markets are competitive and that each carrier is required to sell all transportation services demanded at any given price. All cost function arguments were treated as exogenous variables, input levels as endogenous. No *a priori* restrictions with respect to homotheticity and homogeneity were imposed.

Table 5.1: List of Models Testing Cluster Structures

group	model	testing
G.295	M278.D	M.63 = M.215
G.295	M215.D	M.67 = M.148
G.877	M853.D	M.66 = M.787
G.877	M787.D	M.51 = M.736
G.877	M736.D	M.39 = M.687
G.877	M687.D	M.44 = M.653

Each estimating model consisted of the translog approximation of $C = C[y,w,d]$ around the sample mean, and the derived six or seven factor share equations, depending on the sample being analyzed. Prices for the seven aggregates were constructed following the methodology described in Chapter II, and the derivation of such indices is presented in Appendix B.

The translog form of the cost functions is given by

$$\begin{aligned} \ln C = {} & \alpha_0 + \mathrm{A}_0 d + (\alpha_y + \mathrm{A}_y d) \ln y + (\delta_{yy} + \Delta_{yy} d) (\ln y)^2 + \\ & + \Sigma_k (\beta_k + \mathrm{B}_k d) \ln w_k + 1/2 \Sigma_k \Sigma_l (\gamma_{kl} + \Gamma_{kl} d) \ln w_k \ln w_l + \\ & + \Sigma_k (\rho_{yk} + \mathrm{P}_{yk} d) \ln y \ln w_k \end{aligned} \qquad [5.1]$$

with the share equations in the form

$$\begin{aligned}
S_{\mathrm{O1}} &= \beta_{\mathrm{O1}} + \mathrm{B}_{\mathrm{O1}}\, d + \textstyle\sum_l (\gamma_{\mathrm{O1}l} + \Gamma_{\mathrm{O1}l}\, d) \ln w_l + (\rho_{\mathrm{YO1}} + \mathrm{P}_{\mathrm{YO1}}\, d) \ln y, \\
S_{\mathrm{O2}} &= \beta_{\mathrm{O2}} + \mathrm{B}_{\mathrm{O2}}\, d + \textstyle\sum_l (\gamma_{\mathrm{O2}l} + \Gamma_{\mathrm{O2}l}\, d) \ln w_l + (\rho_{\mathrm{YO2}} + \mathrm{P}_{\mathrm{YO2}}\, d) \ln y, \\
S_{\mathrm{O3}} &= \beta_{\mathrm{O3}} + \mathrm{B}_{\mathrm{O3}}\, d + \textstyle\sum_l (\gamma_{\mathrm{O3}l} + \Gamma_{\mathrm{O3}l}\, d) \ln w_l + (\rho_{\mathrm{YO3}} + \mathrm{P}_{\mathrm{YO3}}\, d) \ln y, \\
S_{\mathrm{O4}} &= \beta_{\mathrm{O4}} + \mathrm{B}_{\mathrm{O4}}\, d + \textstyle\sum_l (\gamma_{\mathrm{O4}l} + \Gamma_{\mathrm{O4}l}\, d) \ln w_l + (\rho_{\mathrm{YO4}} + \mathrm{P}_{\mathrm{YO4}}\, d) \ln y, \qquad [5.2] \\
S_{\mathrm{A4}} &= \beta_{\mathrm{A4}} + \mathrm{B}_{\mathrm{A4}}\, d + \textstyle\sum_l (\gamma_{\mathrm{A4}l} + \Gamma_{\mathrm{A4}l}\, d) \ln w_l + (\rho_{\mathrm{YA4}} + \mathrm{P}_{\mathrm{YA4}}\, d) \ln y, \\
S_{\mathrm{K1}} &= \beta_{\mathrm{K1}} + \mathrm{B}_{\mathrm{K1}}\, d + \textstyle\sum_l (\gamma_{\mathrm{K1}l} + \Gamma_{\mathrm{K1}l}\, d) \ln w_l + (\rho_{\mathrm{YK1}} + \mathrm{P}_{\mathrm{YK1}}\, d) \ln y, \\
S_{\mathrm{E1}} &= \beta_{\mathrm{E1}} + \mathrm{B}_{\mathrm{E1}}\, d + \textstyle\sum_l (\gamma_{\mathrm{E1}l} + \Gamma_{\mathrm{E1}l}\, d) \ln w_l + (\rho_{\mathrm{YE1}} + \mathrm{P}_{\mathrm{YE1}}\, d) \ln y,
\end{aligned}$$

where k, l = O1, O2, O3, O4, A4, K1, and E1, and d is a binary variable taking the values 0 or 1 according to group membership. Clearly, in the case of group G.877, the system did not include the share equation nor the price of the independent trucker (O2).

Symmetry and linear homogeneity in prices were enforced. From the results in Chapter II, these conditions were attained through the parametric restrictions

$$\begin{aligned}
&\gamma_{kl} = \gamma_{lk} \text{ and } \Gamma_{kl} = \Gamma_{lk}, \quad \forall\, k \text{ and } l, \\
&\textstyle\sum_k \beta_k = 1 \text{ and } \sum_k \mathrm{B}_k = 0, \qquad [5.3] \\
&\textstyle\sum_l \gamma_{kl} = \sum_k \gamma_{kl} = \sum_l \Gamma_{kl} = \sum_k \Gamma_{kl} = 0, \\
&\textstyle\sum_k \rho_{yk} = \sum_k \mathrm{P}_{yk} = 0.
\end{aligned}$$

The main hypothesis is that firms within a cluster are technically different from firms outside that cluster. Given this characterization of carrier's technology, should clustering have no effect, all dummy related parameters appearing in the cost model would equal zero:

$$\mathrm{A}_0 = \mathrm{A}_y = \Delta_{yy} = \mathrm{B}_k = \Gamma_{kl} = \mathrm{P}_{yk} = 0, \quad \forall\, k \text{ and } l. \qquad [5.4]$$

Following the terminology introduced in Archibald and Brandt [1987], the total change in factor share as a result of clustering can be decomposed into two main sources: an exogenous bias and an output and price induced bias. Under non-homotheticity, the hypothesis of no factor biased differences is given by

$$B_k = \Gamma_{kl} = P_{yk} = 0, \qquad \forall\ k \text{ and } l, \qquad [5.5]$$

and the hypothesis of no induced price or output factor share bias, but containing a biased clustering effect, is translated into

$$\Gamma_{kl} = P_{yk} = 0, \qquad \forall\ k \text{ and } l. \qquad [5.6]$$

If these no effect hypotheses are rejected, the coefficients Γ_{kl} and P_{yk} will reveal the factor-using/factor-saving nature of the technical differences. The interpretation of these parameters is made clear by noting that each parameter in the cost share equations represents the partial logarithmic derivative of the corresponding input share with respect to either output levels or input prices. Thus, a positive (negative) value for P_{yk} implies that non-homotheticity is factor k using (saving). Similarly, the value of Γ_{kl} measures the extent to which factor share bias is induced by changes in relative factor prices.

5.1.2 Empirical Results

All six models consisted of the cost function [5.1] and the factor share equations [5.2] with the constraints [5.3]. Stochastic disturbances were appended to each equation and assumed to be normally distributed and uncorrelated across firms, but correlated across equations. Zellner's Seemingly Unrelated Regression technique was used. The estimation was carried out using the procedure SYSLIN in SAS, after dropping the *other materials* share equation

(S_{E1}) to avoid singularity of the estimated contemporaneous covariance matrix.[3,4] All hypotheses were evaluated using the test option implemented in the procedure SYSLIN.[5]

The parameter estimates for each cost model are reported in Table C.1 through Table C.18 of Appendix C since, for the current analysis, coefficient estimates are of less interest than are the hypotheses involving the changes in those coefficients across groups. Table 5.2 contains the test statistics for the five hypotheses involving the two models of group G.295. Those involving the models of group G.877 are displayed in Table 5.3. All hypotheses concerning homotheticity and returns to scale, and the other three concerning cluster differences are unequivocally rejected, given the magnitude of the *F* statistics.

Table 5.2: Test Statistics for G.295

model	null hypothesis	test statistic	*p*-value
M278.D	homotheticity	$F^{12}_{1874} = 22.6928$	0.0001
	homogeneity	$F^{14}_{1874} = 19.7538$	0.0001
	no cluster difference	$F^{36}_{1874} = 23.3728$	0.0001
	no factor biased difference	$F^{33}_{1874} = 25.0413$	0.0001
	no induced difference	$F^{27}_{1874} = 8.9217$	0.0001
M215.D	homotheticity	$F^{12}_{1433} = 19.5076$	0.0001
	homogeneity	$F^{14}_{1433} = 16.9886$	0.0001
	no cluster difference	$F^{36}_{1433} = 24.6199$	0.0001
	no factor biased difference	$F^{33}_{1433} = 26.7775$	0.0001
	no induced difference	$F^{27}_{1433} = 7.7276$	0.0001

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[3] The estimates are invariant to which equation is deleted.

[4] Version 5.16 and 5.18 of SAS on an IBM 3090.

[5] Refer to SAS Institute Inc. [1985b] for details in the implementation of these tests.

Table 5.3: Test Statistics for G.877

model	null hypothesis	test statistic	p-value
M853.D	homotheticity	$F^{10}_{5062} = 32.5100$	0.0001
	homogeneity	$F^{12}_{5062} = 32.0311$	0.0001
	no cluster difference	$F^{28}_{5062} = 29.3792$	0.0001
	no factor biased difference	$F^{25}_{5062} = 32.8305$	0.0001
	no induced difference	$F^{20}_{5062} = 8.5839$	0.0001
M787.D	homotheticity	$F^{10}_{4666} = 33.6483$	0.0001
	homogeneity	$F^{12}_{4666} = 32.1893$	0.0001
	no cluster difference	$F^{28}_{4666} = 20.9900$	0.0001
	no factor biased difference	$F^{25}_{4666} = 18.7484$	0.0001
	no induced difference	$F^{20}_{4666} = 6.6272$	0.0001
M736.D	homotheticity	$F^{10}_{4360} = 30.8251$	0.0001
	homogeneity	$F^{12}_{4360} = 30.0759$	0.0001
	no cluster difference	$F^{28}_{4360} = 8.1620$	0.0001
	no factor biased difference	$F^{25}_{4360} = 8.8090$	0.0001
	no induced difference	$F^{20}_{4360} = 2.4962$	0.0002
M697.D	homotheticity	$F^{10}_{4126} = 32.9018$	0.0001
	homogeneity	$F^{12}_{4126} = 32.8014$	0.0001
	no cluster difference	$F^{28}_{4126} = 13.7727$	0.0001
	no factor biased difference	$F^{25}_{4126} = 14.2128$	0.0001
	no induced difference	$F^{20}_{4126} = 5.2708$	0.0001

The direction of the non-homotheticity effects on factor shares is given by the sign of the coefficients P_{yk} shown in Table 5.4. For models M278.D and M215.D, describing the differentiated structure of clusters 5.1 and 5.4 with respect to their parent sample, the results indicate the factor-using output effect associated with the independent trucker, and factor-saving output effects associated with fuel, labor, and capital. This pattern confirms the usually hypothesized conjecture of a technology that balances an internal process using fuel, labor, and capital, with an external one, the independent trucker, incorporating the same factors.

Table 5.4: Effect of Non-Homotheticity on the Use of Inputs

effect	M278.D	M215.D	M853.D	M787.D	M736.D	M697.D
P_{YO1}	⋯	⋯	–	⋯	⋯	⋯
P_{YO2}	+	+	*n.a.*	*n.a.*	*n.a.*	*n.a.*
P_{YO3}	–	–	–	⋯	⋯	⋯
P_{YO4}	–	–	⋯	⋯	⋯	⋯
P_{YA4}	–	⋯	⋯	⋯	⋯	⋯
P_{YK1}	⋯	–	+	–	⋯	+
P_{YE1}	⋯	⋯	⋯	⋯	⋯	⋯

Note: '+' indicates a factor-using effect, '–' a factor-saving effect, and '⋯' a neutral effect;
'*n.a.*' indicates that the effect is not applicable to the model.

The effect of price changes on the least cost combination of production factors is associated with the parameters γ_{kl} and Γ_{kl}. Thus, the differentiated effects between clusters depend on the estimates of Γ_{kl}. The direction of these effects is summarized in Table 5.5.

Table 5.5: Effect of Relative Factor Prices on Factor Bias

effect	M278.D	M215.D	M853.D	M787.D	M736.D	M697.D
Γ_{O1O1}	–	–	–	–	···	–
Γ_{O1O2}	···	–	*n.a.*	*n.a.*	*n.a.*	*n.a.*
Γ_{O1O3}	+	+	···	···	···	···
Γ_{O1O4}	···	···	+	+	···	···
Γ_{O1A4}	···	···	···	···	···	···
Γ_{O1K1}	···	···	+	···	···	···
Γ_{O1E1}	···	···	···	+	···	···
Γ_{O2O2}	+	+	*n.a.*	*n.a.*	*n.a.*	*n.a.*
Γ_{O2O3}	–	–	*n.a.*	*n.a.*	*n.a.*	*n.a.*
Γ_{O2O4}	–	–	*n.a.*	*n.a.*	*n.a.*	*n.a.*
Γ_{O2A4}	–	···	*n.a.*	*n.a.*	*n.a.*	*n.a.*
Γ_{O2K1}	–	–	*n.a.*	*n.a.*	*n.a.*	*n.a.*
Γ_{O2E1}	···	···	*n.a.*	*n.a.*	*n.a.*	*n.a.*
Γ_{O3O3}	···	···	–	–	···	···
Γ_{O3O4}	+	+	···	+	···	···
Γ_{O3A4}	···	···	···	···	···	···
Γ_{O3K1}	+	···	···	+	···	···
Γ_{O3E1}	···	···	···	···	···	···
Γ_{O4O4}	···	···	···	–	–	···
Γ_{O4A4}	···	+	–	···	···	···
Γ_{O4K1}	···	···	···	–	···	···
Γ_{O4E1}	···	···	–	···	···	···
Γ_{A4A4}	···	···	···	···	···	···
Γ_{A4K1}	···	···	···	–	+	···
Γ_{A4E1}	···	···	···	···	···	···
Γ_{K1K1}	–	···	–	+	–	–
Γ_{K1E1}	+	···	···	–	···	···
Γ_{E1E1}	–	···	···	–	···	–

Note: '+' indicates a factor-using effect, '–' a factor-saving effect, and '···' a neutral effect; '*n.a.*' indicates that the effect is not applicable to the model.

Although an increase in factor *l* share due to a price increase of factor *k* relative to the price of *l* might be expected, resulting therefore, in *k* as *l* factor-using, or a reduction in its share as own price of *k* increases, this will not occur necessarily. The implied technical differences are not only dependent upon their factor-using/factor-saving nature, but on the substitution possibilities of the technology as well.

5.2 MODELLING INDIVIDUAL COST STRUCTURES

The models with dummy variables specified in the last section assumed that the stochastic structures of each cluster pair were the same. Given the rejection of the hypotheses of equality between coefficients, the estimation of individual cost models is usually recommended in order to allow for the error structure to differ across groups. For each group of firms defining a cluster, a translog cost model was estimated using the same form given in equations [5.1] and [5.2] with the restrictions [5.3], without the dummy related parameters. The same behavioral assumptions and estimation methods were maintained.

5.2.1 Empirical Results for Group G.295

For firms in group G.295 five models have been estimated. Although only three of them are of interest, i.e., those reflecting the three clustered groups M.63 (cluster 5.1), M.67 (cluster 5.4) and M.148 (cluster 5.5), the estimation of models M.278 and M.215 was carried out to assess the variability of estimated elasticities among levels of aggregation.

The parameter estimates of the five seven-input models are reported in Appendix D, with the derived estimates of the price elasticities of demand and Allen elasticities of substitution evaluated both at the point of approximation and at the average firm of each group.[6] In general, the estimation provided consistent results. Very few parameters are

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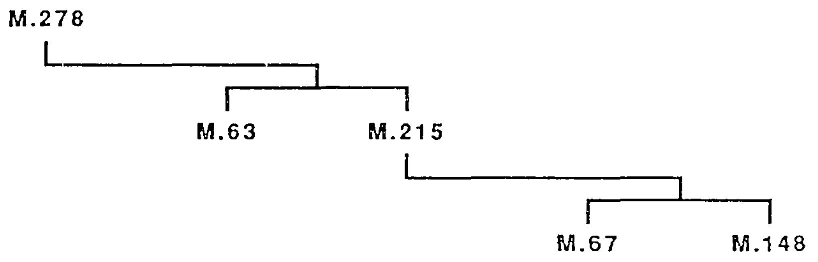
[6] In all cases, the translog systems were estimated around the same point, the overall sample mean, in order to provide a basis for comparing elasticities. To a minor extent, the only affected coefficients are the first-order terms.

statistically non-significant and the R^2's of the cost functions and factor share equations indicate a reasonable goodness of fit. The system-weighted R^2's ranged from 0.66, for model M.278, to 0.75 for model M.67. Homotheticity and homogeneity of the structure of production are rejected hypotheses in all five models.

Table 5.6 displays the own-price elasticities of demand evaluated at the point of approximation. They all have the expected negative sign with the exception of those related to the independent trucker (O2), for models M.63 and M.67, and to capital (K1) in model M.67. Each nonconforming elasticity is, however, essentially zero, indicating that firms in those groups are non-responsive to changes in the price of purchased transportation. This is

Table 5.6: Own-Price Elasticities of Demand - G.295



	M.278	M.63	M.215	M.67	M.148
O1	-0.2785	-0.4928	-0.2694	-0.3462	-0.2470
O2	-0.4137	*0.0314*	-0.5424	*0.0118*	-0.6691
O3	*-0.0301*	-0.5435	-0.0958	-0.2602	-0.1379
O4	-0.3855	-0.4755	-0.3993	-0.5335	-0.2717
A4	-0.3137	-0.4093	-0.3743	-0.4274	-0.4258
K1	-0.1302	-0.5516	-0.0460	*0.0260*	-0.1829
E1	-0.4184	-0.5463	-0.4271	-0.4357	-0.4469

Note: Values in italic indicate that the ratio of estimate to its standard error is smaller than $t(.95,\infty)=1.64$. Standard error of estimates are reported in Tables D.6, D.13, D.20, D.27, and D.34 of Appendix D.

expected given the level of dependence they have on outside capacity.[7] As this dependence decreases, the magnitude of the elasticities increases. It is interesting to note that the elasticities of labor in administration (A4) and of other materials (E1) do not vary substantially between groups, unlike the elasticities of the five other production factors.

Table 5.7 provides estimates of the factor demand and substitution elasticities for the seven factors across the three clusters of interest. In spite of the statistically poor

Table 5.7: Elasticities at the Point of Approximation - G.295

	O1	O2	O3	O4	A4	K1	E1	model
O1	-0.4928	*0.1472*	*0.4787*	2.1303	*0.7004*	1.8060	*0.4500*	M.63
	-0.3462	0.6432	*-0.1574*	0.9923	*-0.5157*	*0.4085*	*0.3544*	M.67
	-0.2470	0.9012	-0.4153	0.6947	*0.5404*	0.6514	0.5056	M.148
O2		*0.0314*	*-0.0550*	-0.7099	*-0.1101*	*-0.0589*	0.5300	M.63
		0.0118	-1.0792	*-0.4217*	*0.6210*	*-0.0880*	0.8247	M.67
		-0.6691	0.8578	0.6322	0.7467	0.7147	0.6846	M.148
O3			-0.5453	3.3596	*1.2210*	2.7319	*0.1390*	M.63
			0.2602	2.1398	*-0.9485*	*0.9456*	*-0.0050*	M.67
			-0.1379	*0.0255*	*0.8136*	*-0.0124*	0.4382	M.148
O4				-0.4755	*0.7314*	1.2148	1.0040	M.63
				-0.5335	*3.3036*	*-0.7730*	*1.0336*	M.67
				-0.2717	*-1.5914*	*0.0982*	0.7352	M.148
A4					-0.4093	*-0.1519*	3.4110	M.63
					-0.4274	*-0.0226*	1.5688	M.67
					-0.4258	*-0.4853*	1.5701	M.148
K1						-0.5516	-0.9670	M.63
						0.0260	-1.5721	M.67
						-0.1829	-0.5122	M.148
E1							-0.5463	M.63
							-0.4397	M.67
							-0.4469	M.148

Note: Diagonal entries are demand elasticities; off-diagonal elements are elasticities of substitution. Values in italic indicate that the ratio of estimate to its standard error is smaller than $t(.95,\infty)=1.64$.

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[7] Recall that for carriers in clusters 5.1 (M.63), expenditures on independent truckers represent about 70 percent of the total cost, while for carriers in cluster 5.4 (M.67) they represent about 38 percent.

estimates involving labor in administration (A4), it is evident that the overall substitution process does indeed change across clusters. For groups M.63 and M.67, for example, the substitution possibilities are very limited. Following an increase in the price of the independent trucker, carriers would respond by cutting their own-capacity activities. This is suggested from the negative sign of the elasticities of substitution on labor in traffic (O4) and fuel (O3) versus the independent trucker, in models M.63 and M.67, respectively. In the case of carriers in M.148, however, there is a substitution process balancing the use of fuel, labor, and capital, with the independent trucker, as verified previously.

Finally, in Table 5.8, the estimates of the returns and economies of scale implied by these cost models are presented. All are statistically significant estimates indicating strong scale economies. Group M.148, composed of the smaller carriers, is the one subject to the highest degree of scale economies, while group M.63, composed of the largest carriers, does not derive the same benefits as output is increased.

Table 5.8: Returns and Economies to Scale Evaluated at the Average Firm

model	returns to scale	economies of scale
M.278	1.5198 –	0.3420 0.02119
M.63	1.1363 –	0.1110 0.05514
M.215	1.5485 –	0.3542 0.02429
M.67	1.2935 –	0.2269 0.03978
M.148	1.6329 –	0.3876 0.03033

Note: standard errors of estimates are indicated in smaller type.

5.2.2 Empirical Results for Group G.877

Estimates for the parameters and elasticities derived for the nine cost functions implied by the structure in Figure 5.3 are reported in Appendix E. Generally, the translog estimates are not as good as those of group G.295. Although systems-weighted R^2's ranged from 0.71 (model M.44) to 0.83 (model M.51), regularity conditions of the cost function were not satisfied within the sample range, given the small number of negative estimated factor shares in some of the models.

The results, however, are very different from those derived for the sample of firms using the independent trucker. Most of the coefficients of the second-order terms estimated for the smaller groups (M.66, M.51, M39, and M.44) are very small in magnitude and not statistically significant, suggesting a Cobb-Douglas type technology, in which the elasticities of substitution between each pair of factors is one.[8] Moreover, homogeneity in output cannot be discarded as an untrue assumption for the structure of production of carriers in groups M.39 and M.44.

Cluster-specific estimates of the own-price elasticities are considerably different from the estimates derived from the full sample model M.853, as indicated in Table 5.9. The same is verified for the estimates of the Allen cross-elastiticies of substitution reported in Table 5.10. For the smaller groups, the elasticities of substitution are close to one, as expected given the essentially zero values for most of the second-order terms of the translog functions.

Although the elasticities differ across the main groups (M.66 through M.653), the estimates derived for the overall sample, that is, for group M.853, are not substantially different from those implied by model M.653. Thus, the consequences of using M.853 to analyze this sector of trucking would not be severe. Nevertheless, the smaller groups do present distinct technological behavior.

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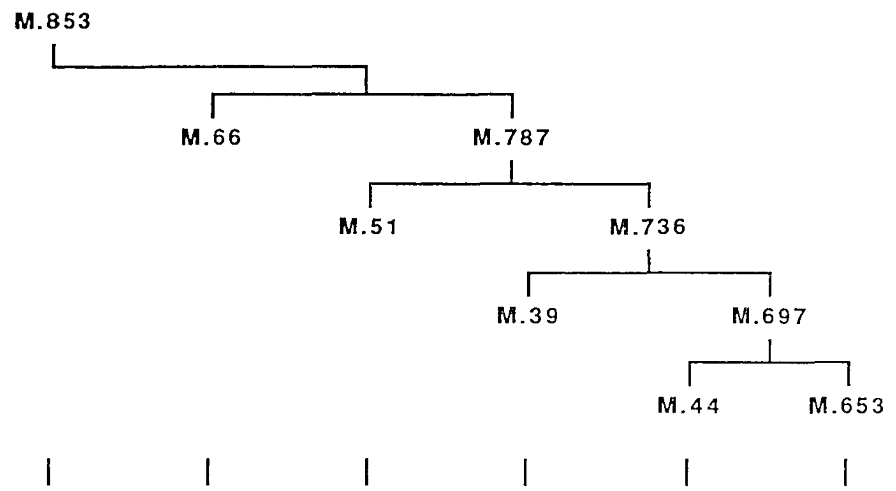
[8] From equation [2.12] of Chapter II, the Allen cross-elasticities are given by $\sigma_{kl} = \gamma_{kl}/S_k S_l + 1$.

Table 5.9: Own-Price Elasticities of Demand - G.877



	M.853	M.66	M.51	M.39	M.44	M.653
O1	-0.2204	-0.3778	-0.3598	-0.1846	-0.4237	-0.2140
O3	-0.2565	-0.5018	-0.5447	-0.2584	-0.3337	-0.2882
O4	*-0.0992*	-0.2370	-0.7662	-0.6876	*-0.2722*	*-0.0476*
A4	*-0.0374*	-0.4937	-0.1267	-0.3590	*-0.6393*	-0.2730
K1	-0.1939	-0.4343	-0.6940	-0.4666	-0.4376	-0.2276
E1	-0.4721	-0.4648	-0.4271	-0.4592	-0.4952	-0.4487

Note: Values in italic indicate that the ratio of estimate to its standard error is smaller than $t(.95,\infty)=1.64$. Standard errors are reported in Tables E.6, E.13, E.27, D.41, E.55 and E.62 of Appendix E.

All groups face very similar increasing returns to scale.[9] According to the results listed in Table 5.11, the degree of returns to scale is about 1.7, with the average firm in group M.66 subject to the highest degree, in the order of 2.5. Comparison with the average firms in G.295 indicates that carriers in G.877 face a much smaller proportional increase in their cost resulting from firm expansion.

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[9] The degree of returns to scale is given by the inverse of the output elasticity $\partial \ln C/\partial \ln y$, and economies of scale measured by $1 - \partial \ln C/\partial \ln y$.

Table 5.10: Elasticities at the Point of Approximation - G.877

	O1	O2	O3	O4	A4	K1	E1	model
O1	-0.3778		-2.1187	1.0963	0.9475	0.7885	1.1502	M.66
	-0.3598		-0.5642	1.1454	0.8535	0.5603	1.2332	M.51
	-0.1846		*-0.0893*	0.9626	*0.4157*	0.3972	0.5788	M.39
	-0.4237		0.6053	0.7790	*-1.3200*	0.8587	0.4910	M.44
	-0.2140		0.2065	*0.0126*	0.3055	0.4621	0.6825	M.653
O2								M.66
								M.51
								M.39
								M.44
								M.653
O3			-0.5018	0.9778	1.2884	0.8485	1.1826	M.66
			-0.5447	1.8602	1.1323	0.9515	0.8902	M.51
			-0.2584	0.7618	0.7417	0.9039	0.6713	M.39
			-0.3337	*0.1579*	1.9540	0.8644	0.5140	M.44
			-0.2882	0.2643	1.0483	0.3881	0.6473	M.653
O4				-0.2370	*-0.0068*	*-0.3911*	*-0.0712*	M.66
				-0.7662	*0.8414*	-0.6394	0.9192	M.51
				-0.6876	*-0.1779*	1.0812	*0.3029*	M.39
				-0.2722	*1.2570*	*-0.5934*	*0.3210*	M.44
				-0.0476	-1.1308	*0.2227*	0.3946	M.653
A4					-0.4937	0.9097	0.4844	M.66
					-0.4703	-0.9729	0.9424	M.51
					-0.3590	*-0.5350*	*0.6792*	M.39
					-0.6393	-3.7705	5.4420	M.44
					-0.2730	*-0.2533*	0.5541	M.653
K1						-0.4343	*-0.0249*	M.66
						-0.1266	-0.0778	M.51
						-0.4666	*-0.0707*	M.39
						-0.4376	-0.8800	M.44
						-0.2275	-0.1774	M.653
E1							-0.4648	M.66
							-0.6941	M.51
							-0.4592	M.39
							-0.4952	M.44
							-0.4487	M.653

Note: Diagonal entries are demand elasticities; off-diagonal elements are elasticities of substitution. Values in italic indicate that the ratio of estimate to its standard error is smaller than $t(.95,\infty)=1.64$.

Table 5.11: Returns and Economies to Scale Evaluated at the Average Firm

model	returns to scale	economies of scale
M.853	1.7923	0.4421
	–	0.01482
M.66	2.4674	0.5947
	–	0.03326
M.787	1.7636	0.4330
	–	0.01580
M.51	1.7264	0.4208
	–	0.03945
M.736	1.7707	0.4352
	–	0.01621
M.39	1.3585	0.2639
	–	0.11582
M.697	1.7888	0.4410
	–	0.01590
M.44	1.7762	0.4370
	–	0.06333
M.653	1.8100	0.4475
	–	0.01622

Note: standard errors of estimates are indicated in smaller type.

5.3 CONCLUSION

As an approximation to a generic cost function, the translog form does not permit a definitive statement that these groups face distinct technologies. The overall conclusion from the results is that the clustering procedure was effective enough to provide a good assignment of firms into groups sharing a common technical behavior; without the characterization of such subsectors, the analysis may have led to erroneous inferences. Of course, as mentioned at the end of Chapter IV, a better assessment of the implications of the results would be achieved if a variety of attributes of trucking operation were available, so they could be used as instruments in the interpretation and validation of a given cluster structure.

In summary, the evidence supports the existence of large differences among firms in the liquid bulk sector and on their market. The economies of scale and responsiveness to input price changes, so distinct between the cluster-determined subsectors, reinforce the initial hypothesis that carriers are strongly regulated by demand requirements.

Chapter VI

CONCLUSIONS AND RECOMMENDATIONS

6.1 OVERVIEW

Data limitations preclude an analysis of the production structure of motor carriers that adequately takes into account the heterogeneity of trucking technology. There is no doubt, however, that such heterogeneity exists and is related to market characteristics; technologies are distinct in terms of transportation services demanded, with respect to both levels and types.

This study attempts to narrow this gap by introducing a methodology to identify similar trucking firms on the basis of their cost share profiles, assuming that market determinants are reflected in the cost allocation process. This methodology is applied to the liquid bulk transport segment of the Brazilian trucking industry using data from 1981. The exploratory phase of the analysis identified two major segments that differ with respect to the use of outside capacity. Within each segment, subgroups also were identified, according to more subtle distinctions in the cost share profile. Detailed analysis using a translog functional form strongly supports the hypothesis of technical differences between the cluster-defined subgroups.

The flexible specification of production employed in this analysis has not only confirmed clustering results and identified the sources of technical differences, but has also demonstrated the inappropriateness of restrictions related to homotheticity of the production structure.

In summary, the methodology developed here goes beyond recent studies that have focused on the production structure of motor carriers. Although some aspects of its implementation may be disputed, for the empirical context of liquid bulk carriage in Brazil, it has been shown that, in the absence of more detailed information about the way transportation services are produced, a formal analysis of the similarity of cost shares is capable of identifying significant differences in production technology. Of course, collection of more detailed data might be preferable, but this often is precluded by financial or institutional considerations, especially in the context of an economy such as Brazil.

6.2 RECOMMENDATIONS

A number of issues remains to be addressed in future research. These basically involve methodological issues regarding the cluster analytic model, the econometric specification, and their interaction with the quality and availability of data, the foremost limiting factor of this project.

Although the IBGE survey has been a good source of information on the road transport sector in Brazil, it is clearly not optimally designed for studies of this type. As discussed in Chapter III, the format of the survey questionnaire is too general, since it has to cover both passenger and freight transport firms. The next step is, therefore, to improve the survey instrument, starting with a better characterization of a firm's output and its interaction with the independent trucker. Minor changes in the questionnaire would provide a more convenient way to characterize subtechnologies and the role of market determinants in the generation of such subtechnologies.

As mentioned at the end of Chapter IV, the preliminary nature of the clustering results must be emphasized. In order to validate the methodology and verify its robustness, external criteria designed to evaluate the extent of recovery of the true cluster structure have to be developed. A particularly constructive approach would be based on the use of the attributes of trucking operation and demand, so they could provide an insight into the interpretation of a given cluster structure. Also, given the heuristic nature of cluster analysis, this would allow the selection of a better suited algorithm to implement the analysis.

With respect to the econometric specification, alternative and perhaps more general and robust specifications could be estimated. For example, forms that possess global properties could be tested, as in the case of the minflex Laurent and the Fourier flexible forms briefly introduced in Chapter II. In spite of the recent theoretical developments in the areas of functional forms and economic aggregation theory, which have extended the boundaries within which technology may be characterized, these boundaries still limit analyses in such a way that empirical work must still be supported by strongly maintained hypotheses.

Appendix A

CONTENTS OF THE IBGE SURVEY

Table A.1: Items in the IBGE Survey

item	description	code
ITEM 01:	QUESTIONNAIRE IDENTIFICATION [a]	
ITEM 02:	FIRM IDENTIFICATION DATA [b]	
ITEM 03:	INVESTMENTS (CR$)	
	in buildings	INV01
	in renovation	INV02
	in new equipments	INV03
	in used equipments	INV04
	in fixtures	INV05
	in furniture and office equipment	INV06
	in new transportation means	INV07
	in used transportation means	INV08
	in concessional rights	INV09
	in financial interests	INV10
ITEM 04:	DIVESTMENTS (CR$)	
	in buildings	DIV01
	in used equipment	DIV02
	in fixtures	DIV03
	in furniture and office equipment	DIV04
	in used transportation equipment	DIV05
	in concessional rights	DIV06
	in financial interests	DIV07

Table A.1: (continued)

item	description	code
ITEM 05:	ASSETS IN 12-31-81 (CR$)	
	land and buildings	NLA01
	machinery and equipments	NLA02
	fixtures	NLA03
	furniture and office equipment	NLA04
	transportation means	NLA05
	in process	NLA06
	concessional rights	NLA07
	financial interests	NLA08
ITEM 06:	PERSONNEL IN 06-30-81	
	owners with activity	LAB01
	administration	LAB02
	traffic	LAB03
	maintenance	LAB04
	other employees	LAB05
	non–paid owners' relatives	LAB06
ITEM 07:	SALARIES, WAGES, AND OTHER REMUNERATIONS (CR$)	
	owners with activity	SAL01
	administration	SAL02
	traffic	SAL03
	maintenance	SAL04
	other	SAL05
	gratuities and profit share	SAL06
ITEM 08:	MONTHLY LABOR FLUCTUATION	
ITEM 09:	DEPRECIATION AND AMORTIZATION (CR$)	
	depreciation and amortization	AMORT
ITEM 10:	GENERAL EXPENSES (CR$)	
	rent and leasing of land and buildings	GEN01
	rent and leasing of office equipment	GEN02
	maintenance of buildings and equipment	GEN03
	advertising	GEN04
	communications	GEN05
	loans and financing of working capital and fixed assets	GEN06
	office supplies and cleaning material	GEN07
	labor related expenses	GEN08

Table A.1: (continued)

item	description	code
ITEM 10:	GENERAL EXPENSES (CR$) - *continued*	
	insurance of buildings and equipment	GEN09
	outside services	GEN10
	utilities (electricity)	GEN11
	miscellaneous	GEN12
ITEM 11:	OPERATING EXPENSES (CR$)	
	vehicle maintenance and parts	OPR01
	printed matter used in traffic	OPR02
	fuel and lubricants	OPR03
	outside vehicle maintenance and repair	OPR04
	terminal fees	OPR05
	licensing	OPR06
	vehicle insurance	OPR07
	purchased capacity	OPR08
	brokerage	OPR09
	indemnities	OPR10
	rent and leasing of trucks	OPR11
	rent and leasing of containers and other equipments	OPR12
	miscellaneous	OPR13
ITEM 12:	REVENUES (CR$) [c]	
	freight urban	REV06
	freight interurban	REV07
	freight interstate	REV08
	freight international	REV09
	freight no fixed routes	REV10
	leasing of vehicles	REV11
	advertising	REV12
	leasing of warehouse, parking, etc.	REV13
	brokerage	REV14
	miscellaneous	REV15
ITEM 13:	NUMBER OF TRAFFIC LINES	
	urban	TRF01
	interurban	TRF02
	interstate	TRF03
	international	TRF04

Table A.1: (continued)

item	description	code
ITEM 14:	EXTENSION OF TRAFFIC LINES (KM)	
	urban	TRF05
	interurban	TRF06
	interstate	TRF07
	international	TRF08
ITEM 15:	PASSENGER OUTPUT [c]	
ITEM 16:	FREIGHT OUTPUT (TONS)	
	urban	CAR01
	interurban	CAR02
	interstate	CAR03
	international	CAR04
	no fixed route	CAR05
ITEM 17:	FLEET COMPOSITION IN 12-31-81[c]	
	trucks	FLT11
	pickups and vans	FLT12
	trailers	FLT13
	piggyback trailers	FLT14
	tractors	FLT19
	towing trucks	FLT20
	other vehicles	FLT21
ITEM 18:	FLEET CAPACITY (TONS)[c]	
	trucks	CAP11
	pickups and vans	CAP12
	trailers	CAP13
	piggyback trailers	CAP14
	other vehicles	CAP21
ITEM 19:	VOLUME OF FUEL AND LUBRICANTS (1000 liters)	
	alcohol	ENR01
	gasoline	ENR02
	diesel	ENR03
	fuel oil (tons)	ENR04
	kerosene	ENR05
	lpg (tons)	ENR06

Table A.1: (continued)

item	description	code
ITEM 20:	EXPENSES WITH FUEL AND LUBRICANTS (CR$)	
	alcohol	ENR11
	gasoline	ENR12
	diesel	ENR13
	fuel oil	ENR14
	kerosene	ENR15
	lpg	ENR16
	other fuels	ENR17
	lubricants	ENR18
ITEM 21:	TAXES (CR$)	
	road transport tax	ISTR
	service tax	ISS
ITEM 22:	NOTES [a]	
ITEM 23:	TAX ID NUMBER [a]	
ITEM 24:	FOR INTERNAL USE [a]	

Note:
a. not available on the data tape.
b. only the beginning and ending dates of the period reflecting the information and the geographical region of firm's headquarters were available on tape.
c. only variables related to liquid bulk freight transport are listed.

Appendix B

DERIVATION OF PRICE INDICES

As discussed in Chapter II, the selection among superlative indices may be viewed as arbitrary. Although Törnquist-Theil's formula has been the one mostly used in recent studies, Fisher's formula was used to derive the price indices of the aggregates mainly because it does not become indeterminate if a price or quantity of a component is zero.

The construction of the price indices for the seven aggregate production factors involved the definition of a reference case. The sample average was taken as the base case.

independent trucker

The index for the price paid for the services of the independent trucker was computed in an unorthodox way since the capacity rented from the owner-operator was not reported in the survey, only the total expenses.

Let x be the transportation output derived from the owner-operator, and let y be the output derived from firm's own capacity. Clearly, $u = x + y$ is firm's total output. If p is the price paid per unit of output of the independent trucker, then $z = p \cdot x$ is the total expenditures with rented capacity. If a function $f(u)$ is defined such that $z = f(u)$, then

$$\frac{\partial f(u)}{\partial x} = \frac{\partial z}{\partial x} = p \qquad [\text{B.1}]$$

is a proxy for the price per ton paid to the independent trucker.

If $u = u(x)$ and $f(u)$ are differentiable functions then

$$\frac{\partial f(u)}{\partial x} = \frac{\partial f(u)}{\partial u} \cdot \frac{\partial u}{\partial x}$$

$$\frac{\partial f(u)}{\partial x} = \frac{\partial f(u)}{\partial (x+y)} \cdot \frac{\partial (x+y)}{\partial x},$$

and using the equality in [B.1]

$$\frac{\partial f(u)}{\partial x} = f'(u)\ [\ 1 + \partial y/\partial x\ ] = p. \qquad \text{[B.2]}$$

Following the notation of Chapter II, the price index is then defined as the ratio

$$\frac{p_1}{p_0} = \frac{f_1'(u)}{f_0'(u)} \cdot \frac{[\ 1 + \partial y/\partial x\ ]_1}{[\ 1 + \partial y/\partial x\ ]_0}, \qquad \text{[B.3]}$$

and, if $\partial y/\partial x$ is assumed to be constant or almost constant in the range of x ($\partial^2 y/\partial x^2 \approx 0$), the second term of [B.3] is approximatelly one, allowing [B.3] to be rewritten as

$$\frac{p_1}{p_0} \approx \frac{f_1'(u)}{f_0'(u)}. \qquad \text{[B.4]}$$

The function $z = f(u)$ was specified in the form $z = \beta_0 \cdot s^{\beta_1} \cdot u^{\beta_2}$, where z is the expenditures with the owner-operator (OPR2), s is its cost share (O2), and u is output

(CARGA). The function was estimated in its linearized (logarithmic) form and the results are given in Tables B.1 and B.2.

The inclusion of O2 introduces simultaneity in the estimation equation, and possibly biases the parameter estimates. However, the model with O2 provided a substantially better fit than that model without it. Also, it is consistent with the clustering hypothesis of product differentiation according to the degree of utilization of a factor input.

Table B.1: Parameter Estimates for OPR2 = *f*(CARGA)

variable [a]	parameter estimate	standard error	*t* statistic	*p*-value
intercept	10.68506370	0.26830200	39.825	0.0001
ln(O2)	1.08404522	0.02138633	50.689	0.0001
ln(CARGA)	0.70417305	0.02623571	26.840	0.0001

Note: a. the equation was estimanted with O2 in the [0,1] interval.

Table B.2: Model Statistics

source	*df*	sum of squares	mean square	*F* statistic	*p*-value
model	2	1895.5003	974.7501	1991.222	0.0001
error	292	138.9815	0.4760		
total	294	2034.4818			
R^2	0.9317				
$\bar{R}^2$	0.9312				

The following result was used to compute the price index between firm 1 and base firm 0, which was that with the average $f'(u)$ computed over the 295 firms in the sample.

$$w_{O2} = \frac{p_1}{p_0} \approx \frac{f_1'(u)}{f_0'(u)} = \frac{[e^{10.6851} \cdot O2^{1.0840} \cdot CARGA^{-0.2958}]_1}{[e^{10.6851} \cdot O2^{1.0840} \cdot CARGA^{-0.2958}]_0}$$

fuel

The *fuel* aggregate is composed of the three basic fuels: alcohol (A), gasoline (G), and diesel (D). Prices for the aggregate components were directly obtained by dividing the total expenditures with each component by the respective volume consumed. That is

$$p_A = \frac{ENR11}{ENR01},\ p_G = \frac{ENR12}{ENR02},\ \text{and } p_D = \frac{ENR13}{ENR03}.$$

Taking the base firm 0 to be that with the average price, the price index of fuel, w_{O3}, was obtained using Fisher's formula given in equation [2.27], and rewritten in [B.5], where the p_i's are the prices defined above and the x_i's are the annual volume consumed of each fuel i, i.e., ENR01, ENR02, and ENR03.

$$w_{O3} = \left[\frac{\sum_i p_i^1 x_i^1}{\sum_i p_i^0 x_i^1} \cdot \frac{\sum_i p_i^1 x_i^0}{\sum_i p_i^0 x_i^0} \right]^{1/2} \text{, for } i = A, G, \text{ and } D. \qquad [B.5]$$

If a firm had not used a given component, its zero price was replaced by the average price computed over all firms that had used that type of fuel.

labor in operation (04) and labor in administration (A4)

Expenditures for labor input within each of the five classes of labor were defined as the sum of payroll plus the cost of all fringe benefits (GEN08) plus gratuities and profit distribution paid (SAL06). These two expenditures were assigned to each class in proportion to the class payroll. The price of labor input in each class was then measured as the class annual labor expenditure per employee.

The composite input *labor in operation* (O4) aggregates personnel in traffic (LAB03) and in maintenance (LAB04), while *labor in administration* (A4) aggregates all other classes of personnel (LAB01, LAB02, and LAB05). Their price indices, w_{O4} and w_{A4} respectively, were computed exactly as the price index of the aggregate fuel w_{O3}.

vehicles

As described in Chapter III, the measure of carrier expenditures with vehicle capital input, KAP1, was defined as the sum of the annualized cost of owning the various types of trucks, assumed to be 14 percent of the value of the vehicle capital stock at the end of the year (NLA05), plus the expenses with vehicle licensing (OPR06), vehicle insurance (OPR07), and the value of rent and leasing payments (OPR11). Since these variables were not disclosed according to vehicle class nor individual truck prices were available, a quantity index for fleet was constructed in order to take into account differences in fleet composition among carriers. The price index of the service of the vehicle capital input, w_{K1}, was then implicitly determined according to the equivalence condition given in [2.28].

The quantity index was derived using Fisher's formula with the assumption that the price of vehicle in class i is proportional to the average carrying capacity of a vehicle in that class. For example, the price of *pickups and vans* was assumed to be proportional to CAP12/FLT12.

Recalling that the quantity index is defined as in [B.5] by interchanging quantities and prices, then q_{K1} was obtained through

$$q_{K1} = \left[\frac{\Sigma_i p_i^1 x_i^1}{\Sigma_i p_i^1 x_i^0} \cdot \frac{\Sigma_i p_i^0 x_i^1}{\Sigma_i p_i^0 x_i^0} \right]^{1/2},$$

where the prices p_i 's were proxied by the average capacity of class i, and the x_i 's were given by the number of trucks in that class. The price index w_{K1} was implicitly obtained using the equivalence condition, i.e.,

$$w_{K1} = \frac{[\text{KAP1}]_1}{[\text{KAP1}]_0} \cdot \frac{1}{q_{K1}}.$$

maintenance and repair

Since the individual quantities of the components of the aggregate *maintenance and repair* were not available and since the amount of maintenance and repair required by a vehicle is expected to be directly proportional to vehicle usage, and therefore, to fuel consumption, the price index for this aggregate was derived assuming quantity of maintenance and repair to be proportional to the total expenditures with fuel, OPR3. That is,

$$w_{O1} = \frac{[\text{OPR1}]_1}{[\text{OPR1}]_0} \cdot \frac{[\text{OPR3}]_0}{[\text{OPR3}]_1}.$$

all other materials

Similarly to the case of maintenance and repair, quantities of the components of the *all other materials* input were not among the reported statistics. Therefore, the amount of the aggregate used was assumed to be proportional to firm size which was proxied by the firm's total carrying capacity, CAPT. Letting ELS1 be the expenditures with all other materials, its price was represented by the ratio ELS1/CAPT, and the price index w_{E1} evaluted as

$$w_{E1} = \frac{[\mathrm{ELS1}]_1}{[\mathrm{ELS1}]_0} \cdot \frac{[\mathrm{CAPT}]_0}{[\mathrm{CAPT}]_1} .$$

Appendix C

TESTING THE EQUALITY OF TRANSLOG COEFFICIENTS BETWEEN CLUSTER STRUCTURES

Table C.0: Testing the Equality between Translog Coefficients

group	model	null hypothesis	test result	tables
G.295	M278.D	M.63 = M.215	reject	C.1 - C.3
G.295	M215.D	M.67 = M.148	reject	C.4 - C.6
G.877	M853.D	M.66 = M.787	reject	C.7 - C.9
G.877	M787.D	M.51 = M.736	reject	C.10 - C.12
G.877	M736.D	M.39 = M.687	reject	C.13 - C.15
G.877	M687.D	M.44 = M.653	reject	C.16 - C.18

Table C.1: Parameter Estimates for M278.D

coefficient	parameter estimate	standard error	t statistic	p-value
α_0	18.31851511	0.05229784	350.273	0.0001
α_Y	0.75550607	0.03577811	21.116	0.0001
δ_{YY}	0.02933253	0.01050128	2.793	0.0058
β_{O1}	0.18634084	0.00515899	36.120	0.0001
β_{O2}	0.27753976	0.00933478	29.732	0.0001
β_{O3}	0.22137415	0.00729584	30.343	0.0001
β_{O4}	0.09078356	0.00523964	17.326	0.0001
β_{A4}	0.04791637	0.00407580	11.756	0.0001
β_{K1}	0.07553426	0.00336804	22.427	0.0001
β_{E1}	0.10051106	0.00436113	23.047	0.0001
1/2 γ_{O1O1}	0.05064043	0.00171685	29.496	0.0001
γ_{O1O2}	-0.00964514	0.00150512	-6.408	0.0001
γ_{O1O3}	-0.06437214	0.00412056	-15.622	0.0001
γ_{O1O4}	-0.00579433	0.00375203	-1.544	0.1242
γ_{O1A4}	-0.00535030	0.00297503	-1.798	0.0737
γ_{O1K1}	-0.00506708	0.00185011	-2.739	0.0068
γ_{O1E1}	-0.01105187	0.00203560	-5.429	0.0001
1/2 γ_{O2O2}	0.02549540	0.00135607	18.801	0.0001
γ_{O2O3}	-0.01433292	0.00213513	-6.713	0.0001
γ_{O2O4}	-0.00854878	0.00150222	-5.691	0.0001
γ_{O2A4}	-0.00374309	0.00115359	-3.245	0.0014
γ_{O2K1}	-0.00662900	0.00085053	-7.794	0.0001
γ_{O2E1}	-0.00809188	0.00127210	-6.361	0.0001
1/2 γ_{O3O3}	0.07538531	0.00488140	15.443	0.0001
γ_{O3O4}	-0.01917364	0.00692365	-2.769	0.0062
γ_{O3A4}	-0.01443814	0.00552889	-2.611	0.0097
γ_{O3K1}	-0.02488249	0.00324313	-7.672	0.0001
γ_{O3E1}	-0.01357130	0.00304258	-4.460	0.0001
1/2 γ_{O4O4}	0.02357415	0.00394671	5.973	0.0001
γ_{O4A4}	-0.00311598	0.00483548	-0.644	0.5201
γ_{O4K1}	-0.00760511	0.00287223	-2.648	0.0088

Table C.1: (continued)

coefficient	parameter estimate	standard error	t statistic	p-value
γ_{O4E1}	-0.00291047	0.00253840	-1.147	0.2530
$1/2\ \gamma_{A4A4}$	0.01364668	0.00277355	4.920	0.0001
γ_{A4K1}	-0.00423963	0.00240610	-1.762	0.0797
γ_{A4E1}	0.00359377	0.00205021	1.753	0.0813
$1/2\ \gamma_{K1K1}$	0.03189768	0.00104807	30.435	0.0001
γ_{K1E1}	-0.01537204	0.00141646	-10.852	0.0001
$1/2\ \gamma_{E1E1}$	0.02370189	0.00113312	20.917	0.0001
ρ_{YO1}	-0.00582059	0.00235927	-2.467	0.0145
ρ_{YO2}	0.02999635	0.00394923	7.595	0.0001
ρ_{YO3}	0.00249387	0.00348695	0.715	0.4754
ρ_{YO4}	0.00139337	0.00252637	0.552	0.5819
ρ_{YA4}	-0.00847438	0.00200569	-4.225	0.0001
ρ_{YK1}	-0.02218042	0.00167908	-13.210	0.0001
ρ_{YE1}	0.00259180	0.00198468	1.306	0.1932
A_0	-0.39846184	0.09546524	-4.174	0.0001
A_Y	0.06607457	0.06343627	1.042	0.2989
Δ_{YY}	0.01958193	0.02388679	0.820	0.4134
B_{O1}	-0.09549317	0.01317924	-7.246	0.0001
B_{O2}	0.22346532	0.03433121	6.509	0.0001
B_{O3}	-0.07241323	0.02745947	-2.637	0.0091
B_{O4}	-0.01627775	0.01405145	-1.158	0.2482
B_{A4}	0.01546343	0.01234168	1.253	0.2118
B_{K1}	-0.00806120	0.00822513	-0.980	0.3283
B_{E1}	-0.04668340	0.00934070	-4.998	0.0001
$1/2\ \Gamma_{O1O1}$	-0.03060758	0.00467752	-6.544	0.0001
Γ_{O1O2}	-0.02479697	0.01524301	-1.627	0.1055
Γ_{O1O3}	0.05491749	0.01264048	4.345	0.0001
Γ_{O1O4}	0.01372451	0.00818875	1.676	0.0954
Γ_{O1A4}	0.00363283	0.00691603	0.525	0.6000
Γ_{O1K1}	0.00645986	0.00427690	1.510	0.1326
Γ_{O1E1}	0.00727743	0.00482145	1.509	0.1329

Table C.1: (continued)

coefficient	parameter estimate	standard error	t statistic	p-value
$1/2\ \Gamma_{O2O2}$	0.11594713	0.02408762	4.814	0.0001
Γ_{O2O3}	-0.08756547	0.04104191	-2.134	0.0342
Γ_{O2O4}	-0.05131677	0.01799699	-2.851	0.0048
Γ_{O2A4}	-0.03805369	0.01698372	-2.241	0.0262
Γ_{O2K1}	-0.02692014	0.00965172	-2.789	0.0058
Γ_{O2E1}	-0.00324122	0.00917060	-0.353	0.7242
$1/2\ \Gamma_{O3O3}$	-0.03309476	0.02467972	-1.341	0.1815
Γ_{O3O4}	0.03827140	0.01698629	2.253	0.0254
Γ_{O3A4}	0.02318305	0.01863436	1.244	0.2150
Γ_{O3K1}	0.03322417	0.00934977	3.553	0.0005
Γ_{O3E1}	0.00415888	0.00788551	0.527	0.5985
$1/2\ \Gamma_{O4O4}$	-0.00709232	0.00748025	-0.948	0.3443
Γ_{O4A4}	0.00064370	0.00985830	0.065	0.9480
Γ_{O4K1}	0.00895207	0.00566311	1.581	0.1156
Γ_{O4E1}	0.00390973	0.00532383	0.734	0.4636
$1/2\ \Gamma_{A4A4}$	0.00358747	0.00672580	0.533	0.5944
Γ_{A4K1}	0.00019124	0.00518166	0.037	0.9706
Γ_{A4E1}	0.00322794	0.00441422	0.731	0.4655
$1/2\ \Gamma_{K1K1}$	-0.01575404	0.00214147	-7.357	0.0001
Γ_{K1E1}	0.00960089	0.00299098	3.210	0.0016
$1/2\ \Gamma_{E1E1}$	-0.01246683	0.00246328	-5.061	0.0001
P_{YO1}	-0.00215892	0.00713575	-0.303	0.7626
P_{YO2}	0.08028882	0.01827216	4.394	0.0001
P_{YO3}	-0.03603269	0.01369147	-2.632	0.0092
P_{YO4}	-0.02631397	0.00811756	-3.242	0.0014
P_{YA4}	-0.01542308	0.00680878	-2.265	0.0246
P_{YK1}	0.00336444	0.00484029	0.695	0.4879
P_{YE1}	-0.00372460	0.00548301	-0.679	0.4978

Note: d = 1 if firm is in M.63 and d = 0 if firm is in M215.

Table C.2: Summary Statistics for M278.D

equation [a]	R^2	*MSE*	*df*
cost	0.8741	0.2859	206
O1	0.7496	0.0024	262
O2	0.8819	0.0085	262
O3	0.7421	0.0047	262
O4	0.4354	0.0022	262
A4	0.2832	0.0013	262
K1	0.6882	0.0010	262
system weighted	0.7659	1.1091	1874

Note: a. statistics for each equation refer to first-stage estimation.

Table C.3: Test Statistics for M278.D

null hypothesis	test statistic	*p*-value
homotheticity	$F^{12}_{1874} = 22.6928$	0.0001
homogeneity	$F^{14}_{1874} = 19.7538$	0.0001
no cluster difference	$F^{36}_{1874} = 23.3728$	0.0001
no factor biased technical difference	$F^{33}_{1874} = 25.0413$	0.0001
induced difference	$F^{27}_{1874} = 8.9217$	0.0001

Table C.4: Parameter Estimates for M215.D

coefficient	parameter estimate	standard error	t statistic	p-value
α_0	18.21093967	0.06387606	285.098	0.0001
α_Y	0.68184874	0.04849955	14.059	0.0001
δ_{YY}	0.01690440	0.01417280	1.193	0.2352
β_{O1}	0.22105827	0.00713143	30.998	0.0001
β_{O2}	0.16047812	0.00684838	23.433	0.0001
β_{O3}	0.26510077	0.00983063	26.967	0.0001
β_{O4}	0.10213968	0.00805170	12.685	0.0001
β_{A4}	0.05051351	0.00640531	7.886	0.0001
β_{K1}	0.08884507	0.00513101	17.315	0.0001
β_{E1}	0.11186457	0.00658894	16.978	0.0001
1/2 γ_{O1O1}	0.05848680	0.00216308	27.039	0.0001
γ_{O1O2}	-0.00343071	0.00157241	-2.182	0.0310
γ_{O1O3}	-0.08487517	0.00507759	-16.716	0.0001
γ_{O1O4}	-0.00513535	0.00501666	-1.024	0.3080
γ_{O1A4}	-0.00571861	0.00390368	-1.465	0.1455
γ_{O1K1}	-0.00580139	0.00241405	-2.403	0.0177
γ_{O1E1}	-0.01201237	0.00246303	-4.877	0.0001
1/2 γ_{O2O2}	0.01368491	0.00084112	16.270	0.0001
γ_{O2O3}	-0.00605598	0.00219870	-2.754	0.0068
γ_{O2O4}	-0.00599385	0.00183467	-3.267	0.0014
γ_{O2A4}	-0.00213634	0.00144467	-1.479	0.1417
γ_{O2K1}	-0.00406538	0.00106701	-3.810	0.0002
γ_{O2E1}	-0.00568755	0.00144194	-3.944	0.0001
1/2 γ_{O3O3}	0.07842847	0.00570272	13.753	0.0001
γ_{O3O4}	-0.02404397	0.00874703	-2.749	0.0069
γ_{O3A4}	-0.00258640	0.00664643	-0.389	0.6978
γ_{O3K1}	-0.02263959	0.00386398	-5.859	0.0001
γ_{O3E1}	-0.01665584	0.00356321	-4.674	0.0001
1/2 γ_{O4O4}	0.03067164	0.00525380	5.838	0.0001
γ_{O4A4}	-0.01329466	0.00622386	-2.136	0.0346
γ_{O4K1}	-0.00964493	0.00368996	-2.614	0.0101

Table C.4: (continued)

coefficient	parameter estimate	standard error	t statistic	p-value
γ_{O4E1}	-0.00323053	0.00308015	-1.049	0.2963
$1/2\ \gamma_{A4A4}$	0.01346995	0.00350907	3.839	0.0002
γ_{A4K1}	-0.00642634	0.00302919	-2.121	0.0359
γ_{A4E1}	0.00322245	0.00247466	1.302	0.1952
$1/2\ \gamma_{K1K1}$	0.03179370	0.00134449	23.647	0.0001
γ_{K1E1}	-0.01500977	0.00176740	-8.493	0.0001
$1/2\ \gamma_{E1E1}$	0.02468681	0.00140117	17.619	0.0001
ρ_{YO1}	-0.00216748	0.00298087	-0.727	0.4685
ρ_{YO2}	0.01337083	0.00263973	5.065	0.0001
ρ_{YO3}	0.00897937	0.00429117	2.093	0.0384
ρ_{YO4}	0.00369912	0.00342128	1.081	0.2817
ρ_{YA4}	-0.00816417	0.00275228	-2.966	0.0036
ρ_{YK1}	-0.01867949	0.00229408	-8.142	0.0001
ρ_{YE1}	0.00296182	0.00272962	1.085	0.2800
A_0	0.01672201	0.10110384	0.165	0.8689
A_Y	0.16782653	0.07023705	2.389	0.0184
Δ_{YY}	0.03596688	0.02161922	1.664	0.0987
B_{O1}	-0.06773831	0.00995692	-6.803	0.0001
B_{O2}	0.23669305	0.01007673	23.489	0.0001
B_{O3}	-0.10148757	0.01405336	-7.222	0.0001
B_{O4}	-0.01916486	0.01109604	-1.727	0.0866
B_{A4}	-0.00129372	0.00888613	-0.146	0.8845
B_{K1}	-0.02402033	0.00738455	-3.253	0.0015
B_{E1}	-0.02298826	0.00954271	-2.409	0.0175
$1/2\ \Gamma_{O1O1}$	-0.01936489	0.00368777	-5.251	0.0001
Γ_{O1O2}	-0.01575845	0.00688894	-2.287	0.0238
Γ_{O1O3}	0.05638284	0.00960251	5.872	0.0001
Γ_{O1O4}	0.00112195	0.00818881	0.137	0.8912
Γ_{O1A4}	-0.00132858	0.00663031	-0.200	0.8415
Γ_{O1K1}	-0.00062609	0.00440603	-0.142	0.8872
Γ_{O1E1}	-0.00106188	0.00503007	-0.211	0.8331

Table C.4: (continued)

coefficient	parameter estimate	standard error	t statistic	p-value
1/2 Γ_{O2O2}	0.10506171	0.01018692	10.313	0.0001
Γ_{O2O3}	-0.12178892	0.02034378	-5.987	0.0001
Γ_{O2O4}	-0.04064513	0.01195625	-3.399	0.0009
Γ_{O2A4}	-0.01175295	0.01093590	-1.075	0.2846
Γ_{O2K1}	-0.02074725	0.00715948	-2.898	0.0044
Γ_{O2E1}	0.00056927	0.00625597	0.091	0.9276
1/2 Γ_{O3O3}	0.01859744	0.01532465	1.214	0.2272
Γ_{O3O4}	0.03505412	0.01615400	2.170	0.0319
Γ_{O3A4}	-0.02224322	0.01498674	-1.484	0.1403
Γ_{O3K1}	0.01340200	0.00900180	1.489	0.1391
Γ_{O3E1}	0.00199830	0.00830817	0.241	0.8103
1/2 Γ_{O4O4}	-0.01714709	0.00885161	-1.937	0.0550
Γ_{O4A4}	0.03192103	0.01133298	2.817	0.0056
Γ_{O4K1}	0.00181030	0.00712140	0.254	0.7998
Γ_{O4E1}	0.00503191	0.00659901	0.763	0.4472
1/2 Γ_{A4A4}	-0.00148389	0.00673329	-0.220	0.8259
Γ_{A4K1}	0.00370046	0.00623627	0.593	0.5540
Γ_{A4E1}	0.00267103	0.00547949	0.487	0.6268
1/2 Γ_{K1K1}	0.00127505	0.00290730	0.439	0.6617
Γ_{K1E1}	-0.00008952	0.00405678	-0.022	0.9824
1/2 Γ_{E1E1}	-0.00455956	0.00316297	-1.442	0.1519
P_{YO1}	-0.00045055	0.00546868	-0.082	0.9345
P_{YO2}	0.07696486	0.00850387	9.051	0.0001
P_{YO3}	-0.05085832	0.00938739	-5.418	0.0001
P_{YO4}	-0.01473044	0.00696438	-2.115	0.0364
P_{YA4}	-0.00015544	0.00586590	-0.026	0.9789
P_{YK1}	-0.01294227	0.00479750	-2.698	0.0079
P_{YE1}	0.00217215	0.00514373	0.422	0.6735

Note: $d = 1$ if firm is in M.67 and $d = 0$ if firm is in M.148.

Table C.5: Summary Statistics for M215.D

equation [a]	R^2	*MSE*	*df*
cost	0.8905	0.2622	143
O1	0.7221	0.0021	199
O2	0.9221	0.0020	199
O3	0.7192	0.0040	199
O4	0.3834	0.0023	199
A4	0.2657	0.0015	199
K1	0.6340	0.0012	199
system weighted	0.8071	1.1262	1433

Note: a. statistics for each equation refer to first-stage estimation.

Table C.6: Test Statistics for M215.D

null hypothesis	test statistic	*p*-value
homotheticity	$F^{12}_{1433} = 19.5076$	0.0001
homogeneity	$F^{14}_{1433} = 16.9886$	0.0001
no cluster difference	$F^{36}_{1433} = 24.6199$	0.0001
no factor biased technical difference	$F^{33}_{1433} = 26.7775$	0.0001
induced difference	$F^{27}_{1433} = 7.7276$	0.0001

Table C.7: Parameter Estimates for M853.D

coefficient	parameter estimate	standard error	t statistic	p-value
α_0	15.85926915	0.02264822	700.244	0.0001
α_Y	0.65433385	0.01792054	36.513	0.0001
δ_{YY}	0.05526497	0.00666604	8.291	0.0001
β_{O1}	0.26129725	0.00257781	101.364	0.0001
β_{O3}	0.30873947	0.00362010	85.285	0.0001
β_{O4}	0.11891204	0.00344722	34.495	0.0001
β_{A4}	0.08447943	0.00323083	26.148	0.0001
β_{K1}	0.14051328	0.00253584	55.411	0.0001
β_{E1}	0.08605853	0.00171859	50.075	0.0001
$1/2\ \gamma_{O1O1}$	0.06886641	0.00108762	63.318	0.0001
γ_{O1O3}	-0.07658539	0.00282383	-27.121	0.0001
γ_{O1O4}	-0.02357855	0.00282703	-8.340	0.0001
γ_{O1A4}	-0.01365475	0.00269459	-5.067	0.0001
γ_{O1K1}	-0.02027689	0.00148403	-13.663	0.0001
γ_{O1E1}	-0.00363723	0.00113395	-3.208	0.0014
$1/2\ \gamma_{O3O3}$	0.06538683	0.00331566	19.721	0.0001
γ_{O3O4}	-0.02218571	0.00519978	-4.267	0.0001
γ_{O3A4}	0.00359197	0.00483770	0.742	0.4580
γ_{O3K1}	-0.02547456	0.00239160	-10.652	0.0001
γ_{O3E1}	-0.01011998	0.00180097	-5.619	0.0001
$1/2\ \gamma_{O4O4}$	0.04631894	0.00334901	13.831	0.0001
γ_{O4A4}	-0.02080388	0.00480553	-4.329	0.0001
γ_{O4K1}	-0.01982267	0.00246159	-8.053	0.0001
γ_{O4E1}	-0.00624707	0.00185395	-3.370	0.0008
$1/2\ \gamma_{A4A4}$	0.02537468	0.00317350	7.996	0.0001
γ_{A4K1}	-0.01602175	0.00239032	-6.703	0.0001
γ_{A4E1}	-0.00386095	0.00180015	-2.145	0.0323
$1/2\ \gamma_{K1K1}$	0.04784834	0.00093526	51.161	0.0001
γ_{K1E1}	-0.01410081	0.00103183	-13.666	0.0001
$1/2\ \gamma_{E1E1}$	0.01898302	0.00054656	34.732	0.0001
ρ_{YO1}	0.01777173	0.00185549	9.578	0.0001

Table C.7: (continued)

coefficient	parameter estimate	standard error	t statistic	p-value
ρ_{YO3}	0.02082729	0.00266814	7.806	0.0001
ρ_{YO4}	-0.00451720	0.00248371	-1.819	0.0693
ρ_{YA4}	-0.00728295	0.00233139	-3.124	0.0019
ρ_{YK1}	-0.03035704	0.00184168	-16.483	0.0001
ρ_{YE1}	0.00355817	0.00121096	2.938	0.0034
A_0	-0.38535964	0.08791243	-4.383	0.0001
A_Y	-0.10631952	0.06487667	-1.639	0.1017
Δ_{YY}	0.00012105	0.01823296	0.007	0.9947
B_{O1}	-0.12393776	0.01110766	-11.158	0.0001
B_{O3}	-0.16950772	0.01692445	-10.016	0.0001
B_{O4}	0.06776189	0.01568957	4.319	0.0001
B_{A4}	0.21604101	0.01688523	12.795	0.0001
B_{K1}	-0.00156892	0.01157177	-0.136	0.8922
B_{E1}	0.01121151	0.00765135	1.465	0.1432
1/2 Γ_{O1O1}	-0.03371879	0.00373864	-9.019	0.0001
Γ_{O1O3}	0.01635955	0.00977724	1.673	0.0947
Γ_{O1O4}	0.02236190	0.00951505	2.350	0.0190
Γ_{O1A4}	0.00683788	0.00968995	0.706	0.4806
Γ_{O1K1}	0.01635176	0.00546626	2.991	0.0029
Γ_{O1E1}	0.00552649	0.00421600	1.311	0.1903
1/2 Γ_{O3O3}	-0.03939913	0.01319956	-2.985	0.0029
Γ_{O3O4}	0.00806347	0.01853101	0.435	0.6636
Γ_{O3A4}	0.02860396	0.02081681	1.374	0.1698
Γ_{O3K1}	0.01342728	0.00973977	1.379	0.1684
Γ_{O3E1}	0.01234399	0.00736797	1.675	0.0943
1/2 Γ_{O4O4}	0.01340217	0.01129429	1.187	0.2357
Γ_{O4A4}	-0.03928555	0.01763612	-2.228	0.0262
Γ_{O4K1}	-0.00407916	0.00918241	-0.444	0.6570
Γ_{O4E1}	-0.01386500	0.00708547	-1.957	0.0507
1/2 Γ_{A4A4}	0.00285588	0.01331710	0.214	0.8303
Γ_{A4K1}	0.00616831	0.00989386	0.623	0.5332

Table C.7: (continued)

coefficient	parameter estimate	standard error	t statistic	p-value
Γ_{A4E1}	-0.00803635	0.00741621	-1.084	0.2789
$1/2\ \Gamma_{K1K1}$	-0.01574985	0.00383829	-4.103	0.0001
Γ_{K1E1}	-0.00036848	0.00430504	-0.086	0.9318
$1/2\ \Gamma_{E1E1}$	0.00219967	0.00224008	0.982	0.3264
P_{YO1}	-0.01291634	0.00595454	-2.169	0.0304
P_{YO3}	-0.02017841	0.00874040	-2.309	0.0212
P_{YO4}	0.01532758	0.00798759	1.919	0.0554
P_{YA4}	-0.00817401	0.00765752	-1.067	0.2861
P_{YK1}	0.01905580	0.00593028	3.213	0.0014
P_{YE1}	0.00688540	0.00394238	1.747	0.0811

Note: $d = 1$ if firm is in M.66 and $d = 0$ if firm is in M.787.

Table C.8: Summary Statistics for M853.D

equation [a]	R^2	*MSE*	*df*
cost	0.7826	0.2398	797
O1	0.7077	0.0032	839
O3	0.5667	0.0063	839
O4	0.2678	0.0055	839
A4	0.5278	0.0047	839
K1	0.6150	0.0032	839
system weighted	0.7604	1.0782	5062

Note: a. statistics for each equation refer to first-stage estimation.

Table C.9: Test Statistics for M853.D

null hypothesis	test statistic	*p*-value
homotheticity	$F^{10}_{5062} = 32.5100$	0.0001
homogeneity	$F^{12}_{5062} = 32.0311$	0.0001
no cluster difference	$F^{28}_{5062} = 29.3792$	0.0001
no factor biased technical difference	$F^{25}_{5062} = 32.8305$	0.0001
induced difference	$F^{20}_{5062} = 8.5839$	0.0001

Table C.10: Parameter Estimates for M787.D

coefficient	parameter estimate	standard error	t statistic	p-value
α_0	15.86148235	0.02313178	685.701	0.0001
α_Y	0.64800687	0.01818856	35.627	0.0001
δ_{YY}	0.05378113	0.00665891	8.077	0.0001
β_{O1}	0.26580378	0.00257154	103.364	0.0001
β_{O3}	0.31273254	0.00374151	83.585	0.0001
β_{O4}	0.12046662	0.00352176	34.206	0.0001
β_{A4}	0.08475029	0.00321945	26.324	0.0001
β_{K1}	0.13096030	0.00223288	58.651	0.0001
β_{E1}	0.08528646	0.00176442	48.337	0.0001
1/2 γ_{O1O1}	0.07101484	0.00112585	63.076	0.0001
γ_{O1O3}	-0.07716686	0.00296625	-26.015	0.0001
γ_{O1O4}	-0.02711044	0.00292302	-9.275	0.0001
γ_{O1A4}	-0.01506230	0.00274261	-5.492	0.0001
γ_{O1K1}	-0.01730006	0.00140554	-12.309	0.0001
γ_{O1E1}	-0.00539001	0.00118152	-4.562	0.0001
1/2 γ_{O3O3}	0.06867402	0.00345163	19.896	0.0001
γ_{O3O4}	-0.02736710	0.00539505	-5.073	0.0001
γ_{O3A4}	0.00197613	0.00497137	0.398	0.6911
γ_{O3K1}	-0.02507934	0.00240085	-10.446	0.0001
γ_{O3E1}	-0.00971086	0.00190794	-5.090	0.0001
1/2 γ_{O4O4}	0.04719891	0.00338479	13.944	0.0001
γ_{O4A4}	-0.02177069	0.00479118	-4.544	0.0001
γ_{O4K1}	-0.01094707	0.00239383	-4.573	0.0001
γ_{O4E1}	-0.00720253	0.00190216	-3.786	0.0002
1/2 γ_{A4A4}	0.02576039	0.00316198	8.147	0.0001
γ_{A4K1}	-0.01294675	0.00231318	-5.597	0.0001
γ_{A4E1}	-0.00371716	0.00182572	-2.036	0.0421
1/2 γ_{K1K1}	0.03990414	0.00083904	47.559	0.0001
γ_{K1E1}	-0.01353506	0.00097948	-13.819	0.0001
1/2 γ_{E1E1}	0.01977781	0.00057140	34.613	0.0001
ρ_{YO1}	0.01685821	0.00184519	9.136	0.0001

Table C.10: (continued)

coefficient	parameter estimate	standard error	t statistic	p-value
ρ_{YO3}	0.02034543	0.00274238	7.419	0.0001
ρ_{YO4}	-0.00689773	0.00252194	-2.735	0.0064
ρ_{YA4}	-0.00787480	0.00230845	-3.411	0.0007
ρ_{YK1}	-0.02560025	0.00161465	-15.855	0.0001
ρ_{YE1}	0.00316913	0.00123719	2.562	0.0106
A_0	-0.02251639	0.09756913	-0.231	0.8176
A_Y	0.04054133	0.07616738	0.532	0.5947
Δ_{YY}	-0.02136562	0.02583132	-0.827	0.4084
B_{O1}	-0.05968429	0.01210082	-4.932	0.0001
B_{O3}	-0.08475000	0.02055337	-4.123	0.0001
B_{O4}	0.00842298	0.02056830	0.410	0.6823
B_{A4}	0.04018725	0.01903876	2.111	0.0351
B_{K1}	0.07543976	0.01375577	5.484	0.0001
B_{E1}	0.02038430	0.00862069	2.365	0.0183
$1/2\ \Gamma_{O1O1}$	-0.02501205	0.00354312	-7.059	0.0001
Γ_{O1O3}	0.00229891	0.00991592	0.232	0.8167
Γ_{O1O4}	0.02315112	0.01016033	2.279	0.0230
Γ_{O1A4}	0.01527931	0.00955460	1.599	0.1102
Γ_{O1K1}	0.00012435	0.00501055	0.025	0.9802
Γ_{O1E1}	0.00917043	0.00379678	2.415	0.0160
$1/2\ \Gamma_{O3O3}$	-0.04600193	0.01359220	-3.384	0.0008
Γ_{O3O4}	0.05705166	0.02189266	2.606	0.0094
Γ_{O3A4}	-0.00604866	0.02006763	-0.301	0.7632
Γ_{O3K1}	0.03350836	0.01034860	3.238	0.0013
Γ_{O3E1}	0.00519359	0.00659617	0.787	0.4313
$1/2\ \Gamma_{O4O4}$	-0.04046372	0.01408167	-2.874	0.0042
Γ_{O4A4}	0.02588803	0.02074518	1.248	0.2125
Γ_{O4K1}	-0.03126355	0.01058105	-2.955	0.0032
Γ_{O4E1}	0.00610018	0.00674680	0.904	0.3662
$1/2\ \Gamma_{A4A4}$	-0.00512121	0.01361576	-0.376	0.7069
Γ_{A4K1}	-0.02825902	0.01061096	-2.663	0.0079

Table C.10: (continued)

coefficient	parameter estimate	standard error	t statistic	p-value
Γ_{A4E1}	0.00338275	0.00658007	0.514	0.6073
$1/2\ \Gamma_{K1K1}$	0.01670216	0.00389354	4.290	0.0001
Γ_{K1E1}	-0.00751444	0.00365014	-2.059	0.0399
$1/2\ \Gamma_{E1E1}$	-0.00816625	0.00176910	-4.616	0.0001
P_{YO1}	-0.00380101	0.00735281	-0.517	0.6054
P_{YO3}	-0.00991908	0.01158041	-0.857	0.3920
P_{YO4}	0.01328620	0.01063478	1.249	0.2120
P_{YA4}	0.00699724	0.00999715	0.700	0.4842
P_{YK1}	-0.01490970	0.00677656	-2.200	0.0281
P_{YE1}	0.00834634	0.00499529	1.671	0.0952

Note: d = 1 if firm is in M.51 and d = 0 if firm is in M.736.

Table C.11: Summary Statistics for M787.D

equation [a]	R^2	MSE	df
cost	0.7862	0.2368	731
O1	0.7213	0.0030	773
O3	0.5201	0.0062	773
O4	0.2447	0.0054	773
A4	0.1482	0.0044	773
K1	0.7346	0.0023	773
system weighted	0.7831	1.0855	4666

Note: a. statistics for each equation refer to first-stage estimation.

Table C.12: Test Statistics for M787.D

null hypothesis	test statistic	p-value
homotheticity	$F^{10}_{4666} = 33.6483$	0.0001
homogeneity	$F^{12}_{4666} = 32.1893$	0.0001
no cluster difference	$F^{28}_{4666} = 20.9900$	0.0001
no factor biased technical difference	$F^{25}_{4666} = 18.7484$	0.0001
induced difference	$F^{20}_{4666} = 6.6272$	0.0001

Table C.13: Parameter Estimates for M736.D

coefficient	parameter estimate	standard error	t statistic	p-value
α_0	15.85958936	0.02334463	679.368	0.0001
α_Y	0.64321892	0.01796217	35.810	0.0001
δ_{YY}	0.05250499	0.00638801	8.219	0.0001
β_{O1}	0.25935308	0.00258025	100.515	0.0001
β_{O3}	0.31594307	0.00397929	79.397	0.0001
β_{O4}	0.12058074	0.00378456	31.861	0.0001
β_{A4}	0.08660027	0.00343561	25.207	0.0001
β_{K1}	0.13289304	0.00221638	59.960	0.0001
β_{E1}	0.08462979	0.00183708	46.068	0.0001
1/2 γ_{O1O1}	0.06585969	0.00115991	56.780	0.0001
γ_{O1O3}	-0.07097655	0.00317987	-22.321	0.0001
γ_{O1O4}	-0.02678622	0.00313876	-8.534	0.0001
γ_{O1A4}	-0.01304272	0.00292955	-4.452	0.0001
γ_{O1K1}	-0.01512345	0.00139629	-10.831	0.0001
γ_{O1E1}	-0.00579045	0.00121350	-4.772	0.0001
1/2 γ_{O3O3}	0.06753879	0.00352807	19.143	0.0001
γ_{O3O4}	-0.03086564	0.00569403	-5.421	0.0001
γ_{O3A4}	0.00291388	0.00518767	0.562	0.5745
γ_{O3K1}	-0.02640591	0.00241197	-10.948	0.0001
γ_{O3E1}	-0.00974335	0.00198954	-4.897	0.0001
1/2 γ_{O4O4}	0.04984061	0.00364287	13.682	0.0001
γ_{O4A4}	-0.02259315	0.00507847	-4.449	0.0001
γ_{O4K1}	-0.01248524	0.00243217	-5.133	0.0001
γ_{O4E1}	-0.00695098	0.00200977	-3.459	0.0006
1/2 γ_{A4A4}	0.02511460	0.00334742	7.503	0.0001
γ_{A4K1}	-0.01452206	0.00233219	-6.227	0.0001
γ_{A4E1}	-0.00298515	0.00191310	-1.560	0.1191
1/2 γ_{K1K1}	0.04103769	0.00080735	50.830	0.0001
γ_{K1E1}	-0.01353872	0.00096282	-14.062	0.0001
1/2 γ_{E1E1}	0.01950432	0.00058899	33.115	0.0001
ρ_{YO1}	0.01538619	0.00180439	8.527	0.0001

Table C.13: (continued)

coefficient	parameter estimate	standard error	*t* statistic	*p*-value
ρ_{YO3}	0.02098217	0.00284814	7.367	0.0001
ρ_{YO4}	-0.00677729	0.00263415	-2.573	0.0103
ρ_{YA4}	-0.00668741	0.00239076	-2.797	0.0053
ρ_{YK1}	-0.02587500	0.00156847	-16.497	0.0001
ρ_{YE1}	0.00297133	0.00126140	2.356	0.0188
A_0	0.08773630	0.09479897	0.925	0.3550
A_Y	-0.04472778	0.10405896	-0.430	0.6675
Δ_{YY}	-0.15489867	0.06616352	-2.341	0.0195
B_{O1}	0.06130619	0.01732672	3.538	0.0004
B_{O3}	-0.03213314	0.02307593	-1.392	0.1642
B_{O4}	-0.01829006	0.02221599	-0.823	0.4106
B_{A4}	-0.01469521	0.01998067	-0.735	0.4623
B_{K1}	-0.00857662	0.01091818	-0.786	0.4324
B_{E1}	0.01238884	0.00921479	1.344	0.1793
$1/2\ \Gamma_{O1O1}$	0.00207306	0.00699143	0.297	0.7669
Γ_{O1O3}	-0.00324585	0.01635440	-0.198	0.8427
Γ_{O1O4}	0.01078844	0.01518848	0.710	0.4778
Γ_{O1A4}	0.00365107	0.01469561	0.248	0.8039
Γ_{O1K1}	-0.00938292	0.00640129	-1.466	0.1432
Γ_{O1E1}	-0.00595687	0.00548897	-1.085	0.2782
$1/2\ \Gamma_{O3O3}$	0.00307328	0.01423932	0.216	0.8292
Γ_{O3O4}	0.02236725	0.02354297	0.950	0.3424
Γ_{O3A4}	-0.04111522	0.02169954	-1.895	0.0586
Γ_{O3K1}	0.00158231	0.00936604	0.169	0.8659
Γ_{O3E1}	0.01426496	0.00828276	1.722	0.0855
$1/2\ \Gamma_{O4O4}$	-0.03529224	0.01558746	-2.264	0.0239
Γ_{O4A4}	0.02422151	0.02254172	1.075	0.2830
Γ_{O4K1}	0.01367500	0.00977048	1.400	0.1621
Γ_{O4E1}	-0.00046771	0.00832678	-0.056	0.9552
$1/2\ \Gamma_{A4A4}$	0.00168306	0.01485308	0.113	0.9098
Γ_{A4K1}	0.01978539	0.00943327	2.097	0.0363

Table C.13: (continued)

coefficient	parameter estimate	standard error	t statistic	p-value
Γ_{A4E1}	-0.00990886	0.00790768	-1.253	0.2106
$1/2\ \Gamma_{K1K1}$	-0.01165313	0.00299825	-3.887	0.0001
Γ_{K1E1}	-0.00235351	0.00393808	-0.598	0.5503
$1/2\ \Gamma_{E1E1}$	0.00221099	0.00236946	0.933	0.3511
P_{YO1}	0.00000699	0.01120521	0.001	0.9995
P_{YO3}	0.00506941	0.01690549	0.300	0.7644
P_{YO4}	-0.00155136	0.01582009	-0.098	0.9219
P_{YA4}	-0.01919828	0.01457197	-1.317	0.1881
P_{YK1}	0.00499042	0.00928960	0.537	0.5913
P_{YE1}	0.01068283	0.00757590	1.410	0.1590

Note: $d = 1$ if firm is in M.39 and $d = 0$ if firm is in M.697.

Table C.14: Summary Statistics for M736.D

equation [a]	R^2	*MSE*	*df*
cost	0.7932	0.2255	680
O1	0.7491	0.0027	722
O3	0.4853	0.0064	722
O4	0.2366	0.0056	722
A4	0.1423	0.0045	722
K1	0.5867	0.0020	722
system weighted	0.7842	1.1068	4360

Note: a. statistics for each equation refer to first-stage estimation.

Table C.15: Test Statistics for M736.D

null hypothesis	test statistic	*p*-value
homotheticity	$F_{4360}^{10} = 30.8251$	0.0001
homogeneity	$F_{4360}^{12} = 30.0759$	0.0001
no cluster difference	$F_{4360}^{28} = 8.1620$	0.0001
no factor biased technical difference	$F_{4360}^{25} = 8.8090$	0.0001
induced difference	$F_{4360}^{20} = 2.4962$	0.0002

Table C.16: Parameter Estimates for M697.D

coefficient	parameter estimate	standard error	t statistic	p-value
α_0	15.83294302	0.02382877	664.447	0.0001
α_Y	0.65012170	0.01820122	35.719	0.0001
δ_{YY}	0.05961285	0.00642649	9.276	0.0001
β_{O1}	0.25625608	0.00259669	98.686	0.0001
β_{O3}	0.30768828	0.00377184	81.575	0.0001
β_{O4}	0.12483000	0.00388461	32.134	0.0001
β_{A4}	0.08940058	0.00347405	25.734	0.0001
β_{K1}	0.13545797	0.00223220	60.684	0.0001
β_{E1}	0.08636708	0.00187401	46.087	0.0001
$1/2\ \gamma_{O1O1}$	0.06773685	0.00122014	55.515	0.0001
γ_{O1O3}	-0.06281049	0.00319788	-19.641	0.0001
γ_{O1O4}	-0.03154375	0.00328125	-9.613	0.0001
γ_{O1A4}	-0.01563747	0.00306073	-5.109	0.0001
γ_{O1K1}	-0.01840919	0.00144678	-12.724	0.0001
γ_{O1E1}	-0.00707280	0.00125556	-5.633	0.0001
$1/2\ \gamma_{O3O3}$	0.06205119	0.00323767	19.165	0.0001
γ_{O3O4}	-0.02832041	0.00548403	-5.164	0.0001
γ_{O3A4}	0.00167470	0.00488994	0.342	0.7321
γ_{O3K1}	-0.02517626	0.00231249	-10.887	0.0001
γ_{O3E1}	-0.00946993	0.00192387	-4.922	0.0001
$1/2\ \gamma_{O4O4}$	0.05164567	0.00376318	13.724	0.0001
γ_{O4A4}	-0.02371688	0.00524007	-4.526	0.0001
γ_{O4K1}	-0.01309041	0.00250345	-5.229	0.0001
γ_{O4E1}	-0.00661990	0.00208910	-3.169	0.0016
$1/2\ \gamma_{A4A4}$	0.02833046	0.00340790	8.313	0.0001
γ_{A4K1}	-0.01559080	0.00239355	-6.514	0.0001
γ_{A4E1}	-0.00339049	0.00196858	-1.722	0.0855
$1/2\ \gamma_{K1K1}$	0.04296364	0.00083277	51.591	0.0001
γ_{K1E1}	-0.01366063	0.00098699	-13.841	0.0001
$1/2\ \gamma_{E1E1}$	0.02010687	0.00061092	32.912	0.0001
ρ_{YO1}	0.01748713	0.00182830	9.565	0.0001

Table C.16: (continued)

coefficient	parameter estimate	standard error	t statistic	p-value
ρ_{YO3}	0.01924074	0.00270561	7.111	0.0001
ρ_{YO4}	-0.00571052	0.00271362	-2.104	0.0357
ρ_{YA4}	-0.00698789	0.00242903	-2.877	0.0042
ρ_{YK1}	-0.02696156	0.00158294	-17.033	0.0001
ρ_{YE1}	0.00293210	0.00129010	2.273	0.0234
A_0	0.33348075	0.09067074	3.678	0.0003
A_Y	-0.13032403	0.07590586	-1.717	0.0865
Δ_{YY}	-0.09602996	0.02888777	-3.324	0.0009
B_{O1}	-0.00590803	0.01445552	-0.409	0.6829
B_{O3}	0.19159186	0.02123014	9.025	0.0001
B_{O4}	-0.05181319	0.02238852	-2.314	0.0210
B_{A4}	-0.05182018	0.02090709	-2.479	0.0135
B_{K1}	-0.04413732	0.01010446	-4.368	0.0001
B_{E1}	-0.03791314	0.00937274	-4.045	0.0001
$1/2\ \Gamma_{O1O1}$	-0.02438639	0.00474742	-5.137	0.0001
Γ_{O1O3}	0.01946857	0.01305661	1.491	0.1364
Γ_{O1O4}	0.02531951	0.01426069	1.775	0.0763
Γ_{O1A4}	-0.00464605	0.01292600	-0.359	0.7194
Γ_{O1K1}	0.00683098	0.00526435	1.298	0.1949
Γ_{O1E1}	0.00179978	0.00508132	0.354	0.7233
$1/2\ \Gamma_{O3O3}$	-0.00786950	0.01669597	-0.471	0.6376
Γ_{O3O4}	0.00116903	0.02692713	0.043	0.9654
Γ_{O3A4}	-0.01046400	0.02308937	-0.453	0.6506
Γ_{O3K1}	0.00429802	0.00903168	0.476	0.6343
Γ_{O3E1}	0.00126739	0.00815725	0.155	0.8766
$1/2\ \Gamma_{O4O4}$	-0.03287519	0.01775853	-1.851	0.0646
Γ_{O4A4}	0.03100411	0.02328680	1.331	0.1835
Γ_{O4K1}	0.00432028	0.00955880	0.452	0.6514
Γ_{O4E1}	0.00393745	0.00907820	0.434	0.6646
$1/2\ \Gamma_{A4A4}$	-0.01838030	0.01478116	-1.243	0.2142
Γ_{A4K1}	0.01140191	0.00875666	1.302	0.1934

Table C.16: (continued)

coefficient	parameter estimate	standard error	t statistic	p-value
Γ_{A4E1}	0.00946464	0.00837786	1.130	0.2590
1/2 Γ_{K1K1}	-0.01407009	0.00274986	-5.117	0.0001
Γ_{K1E1}	0.00128898	0.00357693	0.360	0.7187
1/2 Γ_{E1E1}	-0.00887911	0.00236960	-3.747	0.0002
P_{YO1}	-0.01175676	0.00744102	-1.580	0.1146
P_{YO3}	-0.00736801	0.01152429	-0.639	0.5228
P_{YO4}	-0.00116194	0.01155611	-0.101	0.9199
P_{YA4}	0.00959041	0.01018231	0.942	0.3466
P_{YK1}	0.01318721	0.00633309	2.082	0.0377
P_{YE1}	-0.00249092	0.00529082	-0.471	0.6379

Note: d = 1 if firm is in M.44 and d = 0 if firm is in M.653.

Table C.17: Summary Statistics for M697.D

equation ^{a}	R^2	MSE	df
cost	0.7976	0.2186	641
O1	0.6734	0.0026	683
O3	0.5493	0.0055	683
O4	0.2535	0.0056	683
A4	0.1827	0.0044	683
K1	0.6125	0.0019	683
system weighted	0.7843	1.0877	4126

Note: a. statistics for each equation refer to first-stage estimation.

Table C.18: Test Statistics for M697.D

null hypothesis	test statistic	p-value
homotheticity	$F^{10}_{4126} = 32.9018$	0.0001
homogeneity	$F^{12}_{4126} = 32.8014$	0.0001
no cluster difference	$F^{28}_{4126} = 13.7727$	0.0001
no factor biased technical difference	$F^{25}_{4126} = 14.2128$	0.0001
induced difference	$F^{20}_{4126} = 5.2708$	0.0001

Appendix D

COST MODELS OF GROUP G.295: PARAMETER ESTIMATES AND DERIVED ELASTICITIES

Table D.1: Parameter Estimates for Model M.278

coefficient	parameter estimate	standard error	*t* statistic	*p*-value
α_0	18.27823935	0.04331664	421.968	0.0001
α_Y	0.76834010	0.02887772	26.607	0.0001
δ_{YY}	0.03490712	0.00913039	3.823	0.0002
β_{O1}	0.15310567	0.00477667	32.053	0.0001
β_{O2}	0.38836625	0.01178644	32.950	0.0001
β_{O3}	0.18271237	0.00674699	27.081	0.0001
β_{O4}	0.07615896	0.00415524	18.328	0.0001
β_{A4}	0.04527694	0.00305777	14.807	0.0001
β_{K1}	0.07079275	0.00277042	25.553	0.0001
β_{E1}	0.08358705	0.00357952	23.351	0.0001
$1/2\ \gamma_{O1O1}$	0.04351397	0.00169314	25.700	0.0001
γ_{O1O2}	-0.01690015	0.00163677	-10.325	0.0001
γ_{O1O3}	-0.05022657	0.00404187	-12.427	0.0001
γ_{O1O4}	-0.00022883	0.00333338	-0.069	0.9453
γ_{O1A4}	-0.00503002	0.00268581	-1.873	0.0623
γ_{O1K1}	-0.00428274	0.00178256	-2.403	0.0171
γ_{O1E1}	-0.01035962	0.00192387	-5.385	0.0001

Table D.1: (continued)

coefficient	parameter estimate	standard error	t statistic	p-value
$1/2\ \gamma_{O2O2}$	0.03843121	0.00184052	20.881	0.0001
γ_{O2O3}	-0.02330955	0.00231519	-10.068	0.0001
γ_{O2O4}	-0.01184330	0.00140313	-8.441	0.0001
γ_{O2A4}	-0.00470575	0.00103125	-4.563	0.0001
γ_{O2K1}	-0.00931036	0.00084475	-11.021	0.0001
γ_{O2E1}	-0.01079330	0.00124559	-8.665	0.0001
$1/2\ \gamma_{O3O3}$	0.07191240	0.00494258	14.550	0.0001
γ_{O3O4}	-0.01947670	0.00635098	-3.067	0.0024
γ_{O3A4}	-0.01807638	0.00534076	-3.385	0.0008
γ_{O3K1}	-0.02192541	0.00316447	-6.929	0.0001
γ_{O3E1}	-0.01081019	0.00292209	-3.699	0.0003
$1/2\ \gamma_{O4O4}$	0.02049903	0.00327618	6.257	0.0001
γ_{O4A4}	-0.00261484	0.00415239	-0.630	0.5295
γ_{O4K1}	-0.00553676	0.00252416	-2.194	0.0293
γ_{O4E1}	-0.00129763	0.00222697	-0.583	0.5607
$1/2\ \gamma_{A4A4}$	0.01451297	0.00252465	5.749	0.0001
γ_{A4K1}	-0.00287036	0.00216845	-1.324	0.1869
γ_{A4E1}	0.00427141	0.00180919	2.361	0.0191
$1/2\ \gamma_{K1K1}$	0.02828294	0.00098327	28.764	0.0001
γ_{K1E1}	-0.01264026	0.00132532	-9.537	0.0001
$1/2\ \gamma_{E1E1}$	0.02081479	0.00105553	19.720	0.0001
ρ_{YO1}	-0.00980813	0.00236929	-4.140	0.0001
ρ_{YO2}	0.04918433	0.00531445	9.255	0.0001
ρ_{YO3}	-0.00447740	0.00349048	-1.283	0.2009
ρ_{YO4}	-0.00311958	0.00222933	-1.399	0.1630
ρ_{YA4}	-0.01010586	0.00171473	-5.894	0.0001
ρ_{YK1}	-0.02058782	0.00152219	-13.525	0.0001
ρ_{YE1}	-0.00108555	0.00181603	-0.598	0.5506

Table D.2: Summary Statistics for M.278

equation [a]	R^2	*MSE*	*df*
cost	0.8522	0.2856	242
O1	0.5796	0.0039	270
O2	0.6165	0.0267	270
O3	0.5646	0.0077	270
O4	0.3128	0.0026	270
A4	0.2533	0.0013	270
K1	0.6215	0.0012	270
system weighted	0.6603	1.1387	1910
test for homotheticity	$F^{6}_{1910} = 37.5137$	p-value = 0.0001	
test for homogeneity	$F^{7}_{1910} = 33.1910$	p-value = 0.0001	

Note: a. statistics for each equation refer to first-stage estimation.

Table D.3: Estimated Shares - M.278

evaluated at	O1	O2	O3	O4	A4	K1	E1
average firm [a]	17.0005	27.7865	24.0311	9.1540	5.8036	8.3425	7.8819
	0.3503	0.8536	0.4925	0.2968	0.2176	0.2026	0.2771
point of approx.	15.3106	38.8366	18.2712	7.6159	4.5277	7.0793	8.3587
	0.4777	1.1786	0.6747	0.4155	0.3058	0.2770	0.3580

Note: a. the average firm is defined as the harmonic mean of the cost function variables.
b. standard errors of estimates are indicated in smaller type.

Table D.4: Price Elasticities Evaluated at the Average Shares - M.278

η_{kl}	O1	O2	O3	O4	A4	K1	E1
O1	-0.3181	0.1785	-0.0551	0.0902	0.0285	0.0582	0.0179
	0.01992	0.00963	0.02378	0.01961	0.01580	0.01049	0.01139
O2	0.1092	-0.4455	0.1564	0.0489	0.0411	0.0499	0.0400
	0.00589	0.01325	0.00833	0.00505	0.00371	0.00304	0.00448
O3	-0.0390	0.1809	-0.1612	0.0105	-0.0172	-0.0078	0.0338
	0.01682	0.00963	0.04113	0.02643	0.02222	0.01317	0.01216
O4	0.1675	0.1485	0.0275	-0.4606	0.0295	0.0229	0.0646
	0.03641	0.01533	0.06938	0.07158	0.04536	0.02757	0.02433
A4	0.0833	0.1968	-0.0712	0.0465	-0.4418	0.0340	0.1524
	0.04628	0.01777	0.09202	0.07155	0.08700	0.03736	0.03117
K1	0.1187	0.1663	-0.0225	0.0252	0.0236	-0.2385	-0.0727
	0.02137	0.01013	0.03793	0.03026	0.02599	0.02357	0.01589
E1	0.0386	0.1409	0.1032	0.0751	0.1122	-0.0770	-0.3930
	0.02441	0.01580	0.03707	0.02825	0.02295	0.01681	0.02678

Note: standard errors of estimates are indicated in smaller type.

Table D.5: Elasticities of Substitution Evaluated at the Average Shares

σ_{kl}	O1	O2	O3	O4	A4	K1	E1
O1	-1.8710	0.6422	-0.2294	0.9853	0.4902	0.6980	0.2269
	0.11717	0.03465	0.09893	0.21420	0.27222	0.12569	0.14358
O2	0.6422	-1.6034	0.6509	0.5344	0.7082	0.5984	0.5072
	0.03465	0.04768	0.03467	0.05516	0.06395	0.03644	0.05687
O3	-0.2294	0.6509	-0.6708	0.1146	-0.2961	-0.0937	0.4293
	0.09893	0.03467	0.17117	0.28870	0.38294	0.15785	0.15427
O4	0.9853	0.5344	0.1146	-5.0316	0.5078	0.2750	0.8201
	0.21420	0.05516	0.28870	0.78195	0.78160	0.33053	0.30866
A4	0.4902	0.7082	-0.2961	0.5078	-7.6130	0.4072	1.9338
	0.27222	0.06395	0.38294	0.78160	1.49910	0.44787	0.39551
K1	0.6980	0.5984	-0.0937	0.2750	0.4072	-2.8592	-0.9223
	0.12569	0.03644	0.15785	0.33053	0.44787	0.28256	0.20156
E1	0.2269	0.5072	0.4293	0.8201	1.9338	-0.9223	-4.9863
	0.14358	0.05687	0.15427	0.30866	0.39551	0.20156	0.33982

Note: standard errors of estimates are indicated in smaller type.

Table D.6: Price Elasticities at the Point of Approximation - M.278

η_{kl}	O1	O2	O3	O4	A4	K1	E1
O1	-0.2785	0.2780	-0.1453	0.0747	0.0124	0.0428	0.0159
	0.02212	0.01070	0.02640	0.02177	0.01754	0.01164	0.01257
O2	0.1096	-0.4137	0.1227	0.0457	0.0332	0.0468	0.0558
	0.00421	0.00948	0.00596	0.00361	0.00266	0.00218	0.00321
O3	-0.1218	0.2608	-0.0301	-0.0304	-0.0537	-0.0492	0.0244
	0.02212	0.01267	0.05410	0.03476	0.02923	0.01732	0.01599
O4	0.1501	0.2329	-0.0730	-0.3855	0.0109	-0.0019	0.0666
	0.04377	0.01842	0.08339	0.08604	0.05452	0.03314	0.02924
A4	0.0420	0.2844	-0.2165	0.0184	-0.3137	0.0074	0.1779
	0.05932	0.02278	0.11796	0.09171	0.11152	0.04789	0.03996
K1	0.0926	0.2569	-0.1270	-0.0021	0.0047	-0.1302	-0.0950
	0.02518	0.01193	0.04470	0.03566	0.03063	0.02778	0.01872
E1	0.0292	0.2592	0.0534	0.0606	0.0964	-0.0804	-0.4184
	0.02302	0.01490	0.03496	0.02664	0.02164	0.01586	0.02526

Note: standard errors of estimates are indicated in smaller type.

Table D.7: Elasticities of Substitution at the Point of Approximation

σ_{kl}	O1	O2	O3	O4	A4	K1	E1
O1	-1.8189	0.7158	-0.7955	0.9804	0.2744	0.6049	0.1905
	0.14446	0.02753	0.14449	0.28587	0.38744	0.16446	0.15033
O2	0.7158	-1.0653	0.6715	0.5996	0.7324	0.6614	0.6675
	0.02753	0.02441	0.03263	0.04744	0.05865	0.03073	0.03837
O3	-0.7955	0.6715	-0.1649	-0.3997	-1.1851	-0.6951	0.2922
	0.14449	0.03263	0.29611	0.45641	0.64559	0.24465	0.19133
O4	0.9804	0.5996	-0.3997	-5.0620	0.2417	-0.0269	0.7962
	0.28587	0.04744	0.45641	1.12968	1.20420	0.46817	0.34983
A4	0.2744	0.7324	-1.1851	0.2417	-6.9273	0.1045	2.1286
	0.38744	0.05865	0.64559	1.20420	2.46307	0.67653	0.47804
K1	0.6049	0.6614	-0.6951	-0.0269	0.1045	-1.8388	-1.1361
	0.16446	0.03073	0.24465	0.46817	0.67653	0.39240	0.22397
E1	0.1905	0.6675	0.2922	0.7962	2.1286	-1.1361	-5.0053
	0.15033	0.03837	0.19133	0.34983	0.47804	0.22397	0.30215

Note: standard errors of estimates are indicated in smaller type.

Table D.8: Parameter Estimates for Model M.63

coefficient	parameter estimate	standard error	t statistic	p-value
α_0	17.96594150	0.09039840	198.742	0.0001
α_Y	0.81105404	0.05805981	13.969	0.0001
δ_{YY}	0.02661962	0.02390516	1.114	0.2801
β_{O1}	0.09749679	0.00612528	15.917	0.0001
β_{O2}	0.51049042	0.01742562	29.295	0.0001
β_{O3}	0.13525147	0.01309531	10.328	0.0001
β_{O4}	0.07978656	0.00650677	12.262	0.0001
β_{A4}	0.05757558	0.00666878	8.634	0.0001
β_{K1}	0.06555449	0.00362650	18.077	0.0001
β_{E1}	0.05384469	0.00424330	12.689	0.0001
1/2 γ_{O1O1}	0.01997107	0.00214098	9.328	0.0001
γ_{O1O2}	-0.04244371	0.00786894	-5.394	0.0001
γ_{O1O3}	-0.00687472	0.00628492	-1.094	0.2884
γ_{O1O4}	0.00879282	0.00356362	2.467	0.0239
γ_{O1A4}	-0.00168203	0.00357196	-0.471	0.6434
γ_{O1K1}	0.00515114	0.00191932	2.684	0.0152
γ_{O1E1}	-0.00288563	0.00211764	-1.363	0.1898
1/2 γ_{O2O2}	0.13296249	0.01256999	10.578	0.0001
γ_{O2O3}	-0.07284303	0.01973385	-3.691	0.0017
γ_{O2O4}	-0.06964655	0.00898228	-7.754	0.0001
γ_{O2A4}	-0.03262859	0.00927883	-3.516	0.0025
γ_{O2K1}	-0.03543742	0.00478111	-7.412	0.0001
γ_{O2E1}	-0.01292569	0.00470796	-2.745	0.0133
1/2 γ_{O3O3}	0.02172344	0.01103524	1.969	0.0646
γ_{O3O4}	0.02546339	0.00742035	3.432	0.0030
γ_{O3A4}	0.00172087	0.00873693	0.197	0.8461
γ_{O3K1}	0.01535548	0.00410997	3.736	0.0015
γ_{O3E1}	-0.00626888	0.00364892	-1.718	0.1029
1/2 γ_{O4O4}	0.01774115	0.00304370	5.829	0.0001
γ_{O4A4}	-0.00123402	0.00457652	-0.270	0.7905
γ_{O4K1}	0.00112359	0.00236823	0.474	0.6409

Table D.8: (continued)

coefficient	parameter estimate	standard error	t statistic	p-value
γ_{O4E1}	0.00001848	0.00225024	0.008	0.9935
$1/2\ \gamma_{A4A4}$	0.01534807	0.00335771	4.571	0.0002
γ_{A4K1}	-0.00434766	0.00243255	-1.787	0.0907
γ_{A4E1}	0.00747530	0.00228901	3.266	0.0043
$1/2\ \gamma_{K1K1}$	0.01254865	0.00089131	14.079	0.0001
γ_{K1E1}	-0.00694244	0.00127319	-5.453	0.0001
$1/2\ \gamma_{E1E1}$	0.01076443	0.00102279	10.525	0.0001
ρ_{YO1}	-0.01248842	0.00340886	-3.664	0.0018
ρ_{YO2}	0.10667009	0.00953872	11.183	0.0001
ρ_{YO3}	-0.02821562	0.00664969	-4.243	0.0005
ρ_{YO4}	-0.02787592	0.00383403	-7.271	0.0001
ρ_{YA4}	-0.02091870	0.00392681	-5.327	0.0001
ρ_{YK1}	-0.01633489	0.00216574	-7.542	0.0001
ρ_{YE1}	-0.00083653	0.00265180	-0.315	0.7560

Table D.9: Summary Statistics for M.63

equation [a]	R^2	*MSE*	*df*
cost	0.8823	0.3642	27
O1	0.6335	0.0006	55
O2	0.8208	0.0024	55
O3	0.5880	0.0013	55
O4	0.5063	0.0005	55
A4	0.5090	0.0005	55
K1	0.7197	0.0002	55
system weighted	0.6759	1.2452	405
test for homotheticity	$F^{6}_{405} = 25.2992$	p-value = 0.0001	
test for homogeneity	$F^{7}_{405} = 21.6869$	p-value = 0.0001	

Note: a. statistics for each equation refer to first-stage estimation.

Table D.10: Estimated Shares - M.63

evaluated at	O1	O2	O3	O4	A4	K1	E1
average firm [a]	6.3060	69.4323	8.8390	3.7674	4.0285	3.5061	4.1207
	0.2999	0.6214	0.4510	0.2928	0.2949	0.1758	0.2688
point of approx.	9.7497	51.0490	13.5251	7.9787	5.7576	6.5555	5.3845
	0.6125	1.7426	1.3095	0.6507	0.6669	0.3627	0.4243

Note: a. the average firm is defined as the harmonic mean of the cost function variables.
b. standard errors of estimates are indicated in smaller type.

Table D.11: Price Elasticities Evaluated at the Average Shares - M.63

η_{kl}	O1	O2	O3	O4	A4	K1	E1
O1	-0.3035	0.0213	-0.0206	0.1771	0.0136	0.1168	-0.0046
	0.06790	0.12478	0.09967	0.05651	0.05664	0.03044	0.03358
O2	0.0019	0.0773	-0.0165	-0.0626	-0.0067	-0.0160	0.0226
	0.01133	0.03621	0.02842	0.01294	0.01336	0.00689	0.00678
O3	-0.0147	-0.1298	-0.4201	0.3258	0.0598	0.2088	-0.0297
	0.07110	0.22326	0.24969	0.08395	0.09885	0.04650	0.04128
O4	0.2965	-1.1543	0.7643	-0.0205	0.0075	0.0649	0.0417
	0.09459	0.23842	0.19696	0.16158	0.12148	0.06286	0.05973
A4	0.0213	-0.1156	0.1311	0.0070	-0.1977	-0.0729	0.2268
	0.08867	0.23033	0.21688	0.11360	0.16670	0.06038	0.05682
K1	0.2100	-0.3164	0.5264	0.0697	-0.0837	-0.2491	-0.1568
	0.05474	0.13637	0.11723	0.06755	0.06938	0.05084	0.03631
E1	-0.0070	0.3806	-0.0637	0.0381	0.2217	-0.1334	-0.4363
	0.05139	0.11425	0.08855	0.05461	0.05555	0.03090	0.04964

Note: standard errors of estimates are indicated in smaller type.

Table D.12: Elasticities of Substitution Evaluated at the Average Shares

σ_{kl}	O1	O2	O3	O4	A4	K1	E1
O1	-4.8136	0.0306	-0.2334	4.7011	0.3379	3.3299	-0.1100
	1.07679	0.17972	1.12756	1.50001	1.40608	0.86811	0.81493
O2	0.0306	0.1114	-0.1869	-1.6626	-0.1665	-0.4557	0.5480
	0.17972	0.05215	0.32155	0.34339	0.33174	0.19640	0.16455
O3	-0.2334	-0.1869	-4.7525	8.6467	1.4833	5.9550	-0.7210
	1.12756	0.32155	2.82490	2.22833	2.45366	1.32622	1.00181
O4	4.7011	-1.6626	8.6467	-0.5441	0.1869	1.8506	1.0120
	1.50001	0.34339	2.22833	4.28895	3.01547	1.79294	1.44948
A4	0.3379	-0.1665	1.4833	0.1869	-4.9085	-2.0782	5.5030
	1.40608	0.33174	2.45366	3.01547	4.13801	1.72228	1.37890
K1	3.3299	-0.4557	5.9550	1.8506	-2.0782	-7.1052	-3.8050
	0.86811	0.19640	1.32622	1.79294	1.72228	1.45018	0.88125
E1	-0.1105	0.5482	-0.7211	1.0119	5.5031	-3.8053	-10.5890
	0.81493	0.16455	1.00181	1.44948	1.37890	0.88125	1.20467

Note: standard errors of estimates are indicated in smaller type.

Table D.13: Price Elasticities at the Point of Approximation - M.63

η_{kl}	O1	O2	O3	O4	A4	K1	E1
O1	-0.4928	0.0752	0.0647	0.1700	0.0403	0.1184	0.0243
	0.04392	0.08071	0.06446	0.03655	0.03664	0.01969	0.02172
O2	0.0144	0.0314	-0.0074	-0.0566	-0.0063	-0.0039	0.0285
	0.01541	0.04925	0.03866	0.01760	0.01818	0.00937	0.00922
O3	0.0467	-0.0281	-0.5435	0.2681	0.0703	0.1791	0.0075
	0.04647	0.14591	0.16318	0.05486	0.06460	0.03039	0.02698
O4	0.2077	-0.3624	0.4544	-0.4755	0.0421	0.0796	0.0541
	0.04466	0.11258	0.09300	0.07630	0.05736	0.02968	0.02820
A4	0.0683	-0.0562	0.1651	0.0584	-0.4093	-0.0100	0.1837
	0.06204	0.16116	0.15175	0.07949	0.11664	0.04225	0.03976
K1	0.1761	-0.0301	0.3695	0.0969	-0.0088	-0.5516	-0.0521
	0.02928	0.07293	0.06270	0.03613	0.03711	0.02719	0.01942
E1	0.0439	0.2704	0.0188	0.0801	0.1964	-0.0634	-0.5463
	0.03933	0.08744	0.06777	0.04179	0.04251	0.02365	0.03799

Note: standard errors of estimates are indicated in smaller type.

Table D.14: Elasticities of Substitution at the Point of Approximation

σ_{kl}	O1	O2	O3	O4	A4	K1	E1
O1	-5.0548	0.1472	0.4787	2.1303	0.7004	1.8060	0.4500
	0.45047	0.15810	0.47661	0.45811	0.63632	0.30030	0.40339
O2	0.1472	0.0615	-0.0550	-0.7099	-0.1101	-0.0589	0.5300
	0.15810	0.09647	0.28581	0.22053	0.31569	0.14287	0.17128
O3	0.4787	-0.0550	-4.0186	3.3596	1.2210	2.7319	0.1390
	0.47661	0.28581	1.20650	0.68763	1.12196	0.46355	0.50105
O4	2.1303	-0.7100	3.3596	-5.9596	0.7314	1.2148	1.0040
	0.45811	0.22053	0.68763	0.95625	0.99625	0.45279	0.52379
A4	0.7004	-0.1101	1.2210	0.7314	-7.1086	-0.1519	3.4110
	0.63632	0.31570	1.12196	0.99625	2.02580	0.64450	0.73836
K1	1.8060	-0.0589	2.7319	1.2148	-0.1519	-8.4144	-0.9670
	0.30030	0.14287	0.46355	0.45279	0.64450	0.41481	0.36070
E1	0.4503	0.5298	0.1392	1.0043	3.4113	-0.9668	-10.1460
	0.40339	0.17128	0.50105	0.52379	0.73836	0.36070	0.70556

Note: standard errors of estimates are indicated in smaller type.

Table D.15: Parameter Estimates for Model M.215

coefficient	parameter estimate	standard error	t statistic	p-value
α_0	18.31536444	0.05152826	355.443	0.0001
α_Y	0.75312340	0.03483253	21.621	0.0001
δ_{YY}	0.02873098	0.01001503	2.869	0.0046
β_{O1}	0.18723675	0.00563298	33.239	0.0001
β_{O2}	0.27506956	0.01009776	27.241	0.0001
β_{O3}	0.22272716	0.00794418	28.037	0.0001
β_{O4}	0.09123515	0.00573389	15.912	0.0001
β_{A4}	0.04785291	0.00438278	10.918	0.0001
β_{K1}	0.07517411	0.00369940	20.321	0.0001
β_{E1}	0.10070436	0.00476438	21.137	0.0001
1/2 γ_{O1O1}	0.05086512	0.00187063	27.192	0.0001
γ_{O1O2}	-0.00923481	0.00164033	-5.630	0.0001
γ_{O1O3}	-0.06367701	0.00450198	-14.144	0.0001
γ_{O1O4}	-0.00596186	0.00410777	-1.451	0.1485
γ_{O1A4}	-0.00548181	0.00321411	-1.706	0.0899
γ_{O1K1}	-0.00623142	0.00199800	-3.119	0.0021
γ_{O1E1}	-0.01114333	0.00221934	-5.021	0.0001
1/2 γ_{O2O2}	0.02510301	0.00144435	17.380	0.0001
γ_{O2O3}	-0.01374189	0.00231700	-5.931	0.0001
γ_{O2O4}	-0.00847820	0.00164360	-5.158	0.0001
γ_{O2A4}	-0.00379537	0.00124071	-3.059	0.0026
γ_{O2K1}	-0.00678047	0.00091830	-7.384	0.0001
γ_{O2E1}	-0.00817529	0.00138356	-5.909	0.0001
1/2 γ_{O3O3}	0.07589099	0.00535336	14.176	0.0001
γ_{O3O4}	-0.01868595	0.00758354	-2.464	0.0147
γ_{O3A4}	-0.01548288	0.00600841	-2.577	0.0108
γ_{O3K1}	-0.02660105	0.00353537	-7.524	0.0001
γ_{O3E1}	-0.01359321	0.00332577	-4.087	0.0001
1/2 γ_{O4O4}	0.02324066	0.00430898	5.394	0.0001
γ_{O4A4}	-0.00300629	0.00524782	-0.573	0.5675
γ_{O4K1}	-0.00749761	0.00311541	-2.407	0.0172

Table D.15: (continued)

coefficient	parameter estimate	standard error	t statistic	p-value
γ_{O4E1}	-0.00285140	0.00277641	-1.027	0.3059
$1/2\ \gamma_{A4A4}$	0.01382512	0.00299633	4.614	0.0001
γ_{A4K1}	-0.00352167	0.00259677	-1.356	0.1768
γ_{A4E1}	0.00363777	0.00221524	1.642	0.1024
$1/2\ \gamma_{K1K1}$	0.03303126	0.00113075	29.212	0.0001
γ_{K1E1}	-0.01543031	0.00152664	-10.107	0.0001
$1/2\ \gamma_{E1E1}$	0.02377788	0.00124150	19.152	0.0001
ρ_{YO1}	-0.00502491	0.00256427	-1.960	0.0517
ρ_{YO2}	0.02938878	0.00421776	6.968	0.0001
ρ_{YO3}	0.00366098	0.00378674	0.967	0.3350
ρ_{YO4}	0.00145560	0.00276201	0.527	0.5989
ρ_{YA4}	-0.00884739	0.00215709	-4.102	0.0001
ρ_{YK1}	-0.02322528	0.00183772	-12.638	0.0001
ρ_{YE1}	0.00259223	0.00216150	1.199	0.2321

Table D.16: Summary Statistics for M.215

equation [a]	R^2	*MSE*	*df*
cost	0.8567	0.2741	179
O1	0.6135	0.0029	207
O2	0.5923	0.0101	207
O3	0.5934	0.0056	207
O4	0.2706	0.0026	207
A4	0.2066	0.0015	207
K1	0.6013	0.0012	207
system weighted	0.7238	1.1155	1469
test for homotheticity	$F^{6}_{1469} = 36.7398$	p-value = 0.0001	
test for homogeneity	$F^{7}_{1469} = 31.9997$	p-value = 0.0001	

Note: a. statistics for each equation refer to first-stage estimation.

Table D.17: Estimated Shares - M.215

evaluated at	O1	O2	O3	O4	A4	K1	E1
average firm [a]	19.5717	17.7093	27.6656	10.4363	6.2496	9.5963	8.7712
	0.3574	0.6387	0.4992	0.3474	0.2651	0.2364	0.3233
point of approx.	18.7237	27.5070	22.2727	9.1235	4.7853	7.51741	10.0704
	0.5633	1.0098	0.7944	0.5734	0.4383	0.36994	0.4764

Note: a. the average firm is defined as the harmonic mean of the cost function variables.
b. standard errors of estimates are indicated in smaller type.

Table D.18: Price Elasticities Evaluated at the Average Shares - M.215

η_{kl}	O1	O2	O3	O4	A4	K1	E1
O1	-0.2845	0.1299	-0.0487	0.0739	0.0345	0.0641	0.0308
	0.01912	0.00838	0.02300	0.02099	0.01642	0.01021	0.01134
O2	0.1436	-0.5394	0.1991	0.0565	0.0411	0.0577	0.0416
	0.00926	0.01631	0.01308	0.00928	0.00701	0.00519	0.00781
O3	-0.0345	0.1274	-0.1747	0.0368	0.0065	-0.0002	0.0386
	0.01627	0.00838	0.03870	0.02741	0.02172	0.01278	0.01202
O4	0.1386	0.0959	0.0976	-0.4503	0.0337	0.0241	0.0604
	0.03936	0.01575	0.07267	0.08258	0.05028	0.02985	0.02660
A4	0.1080	0.1164	0.0289	0.0563	-0.4951	0.0396	0.1459
	0.05143	0.01985	0.09614	0.08397	0.09589	0.04155	0.03545
K1	0.1308	0.1064	-0.0005	0.0262	0.0258	-0.2156	-0.0731
	0.02082	0.00957	0.03684	0.03246	0.02706	0.02357	0.01591
E1	0.0687	0.0839	0.1217	0.0719	0.1040	-0.0800	-0.3701
	0.02530	0.01577	0.03792	0.03165	0.02526	0.01741	0.02831

Note: standard errors of estimates are indicated in smaller type.

Table D.19: Elasticities of Substitution Evaluated at the Average Shares

σ_{kl}	O1	O2	O3	O4	A4	K1	E1
O1	-1.4536	0.7336	-0.1760	0.7081	0.5518	0.6682	0.3509
	0.09767	0.04733	0.08315	0.20111	0.26277	0.10638	0.12928
O2	0.7336	-3.0459	0.7195	0.5413	0.6571	0.6010	0.4737
	0.04733	0.09211	0.04729	0.08893	0.11210	0.05404	0.08907
O3	-0.1760	0.7195	-0.6315	0.3528	0.1045	-0.0020	0.4398
	0.08315	0.04729	0.13989	0.26266	0.34751	0.13317	0.13705
O4	0.7081	0.5413	0.3528	-4.3143	0.5391	0.2514	0.6885
	0.20111	0.08893	0.26266	0.79125	0.80461	0.31107	0.30330
A4	0.5518	0.6571	0.1045	0.5391	-7.9217	0.4128	1.6636
	0.26277	0.11210	0.34751	0.80461	1.53434	0.43299	0.40412
K1	0.6682	0.6010	-0.0020	0.2514	0.4128	-2.2469	-0.8332
	0.10638	0.05404	0.13317	0.31107	0.43299	0.24558	0.18137
E1	0.3509	0.4737	0.4398	0.6885	1.6636	-0.8332	-4.2196
	0.12928	0.08907	0.13705	0.30330	0.40412	0.18137	0.32275

Note: standard errors of estimates are indicated in smaller type.

Table D.20: Price Elasticities at the Point of Approximation - M.215

η_{kl}	O1	O2	O3	O4	A4	K1	E1
O1	-0.2694	0.2258	-0.1174	0.0594	0.0186	0.0419	0.0412
	0.01998	0.00876	0.02404	0.02194	0.01717	0.01067	0.01185
O2	0.1537	-0.5424	0.1728	0.0604	0.0341	0.0505	0.0710
	0.00596	0.01050	0.00842	0.00598	0.00451	0.00334	0.00503
O3	-0.0987	0.2134	-0.0958	0.0073	-0.0217	-0.0443	0.0397
	0.02021	0.01040	0.04807	0.03405	0.02698	0.01587	0.01493
O4	0.1219	0.1821	0.0179	-0.3993	0.0149	-0.0070	0.0695
	0.04502	0.01802	0.08312	0.09446	0.05752	0.03415	0.03043
A4	0.0727	0.1958	-0.1008	0.0284	-0.3743	0.0016	0.1767
	0.06717	0.02593	0.12556	0.10967	0.12523	0.05427	0.04629
K1	0.1043	0.1849	-0.1311	-0.0085	0.0010	-0.0460	-0.1046
	0.02658	0.01222	0.04703	0.04144	0.03454	0.03008	0.02031
E1	0.0766	0.1939	0.0878	0.0629	0.0840	-0.0781	-0.4271
	0.02204	0.01374	0.03303	0.02757	0.02200	0.01516	0.02466

Note: standard errors of estimates are indicated in smaller type.

Table D.21: Elasticities of Substitution at the Point of Approximation

σ_{kl}	O1	O2	O3	O4	A4	K1	E1
O1	-1.4390	0.8207	-0.5269	0.6510	0.3882	0.5573	0.4090
	0.10672	0.03185	0.10795	0.24047	0.35872	0.14195	0.11770
O2	0.8207	-1.9719	0.7757	0.6622	0.7117	0.6721	0.7049
	0.03185	0.03818	0.03782	0.06549	0.09426	0.04441	0.04995
O3	-0.5269	0.7757	-0.4301	0.0804	-0.4527	-0.5888	0.3940
	0.10795	0.03782	0.21583	0.37320	0.56374	0.21115	0.14828
O4	0.6510	0.6622	0.0804	-4.3766	0.3114	-0.0932	0.6897
	0.24047	0.06549	0.37320	1.03533	1.20201	0.45424	0.30219
A4	0.3882	0.7117	-0.4527	0.3114	-7.8225	0.0210	1.7549
	0.35873	0.09426	0.56374	1.20201	2.61699	0.72187	0.45969
K1	0.5573	0.6721	-0.5888	-0.0932	0.0210	-0.6123	-1.0383
	0.14195	0.04441	0.21115	0.45424	0.72187	0.40019	0.20166
E1	0.4090	0.7049	0.3940	0.6897	1.7549	-1.0383	-4.2408
	0.11770	0.04995	0.14828	0.30218	0.45969	0.20166	0.24484

Note: standard errors of estimates are indicated in smaller type.

Table D.22: Parameter Estimates for Model M.67

coefficient	parameter estimate	standard error	t statistic	p-value
α_0	18.22143026	0.07738845	235.454	0.0001
α_Y	0.85609481	0.04992844	17.146	0.0001
δ_{YY}	0.05718042	0.01592279	3.591	0.0016
β_{O1}	0.15446677	0.00569454	27.125	0.0001
β_{O2}	0.39732819	0.00554176	71.697	0.0001
β_{O3}	0.16202084	0.00784041	20.665	0.0001
β_{O4}	0.08276961	0.00723899	11.434	0.0001
β_{A4}	0.04778359	0.00523592	9.126	0.0001
β_{K1}	0.06511783	0.00445170	14.628	0.0001
β_{E1}	0.09051316	0.00470714	19.229	0.0001
1/2 γ_{O1O1}	0.03856372	0.00253816	15.194	0.0001
γ_{O1O2}	-0.02189915	0.00492800	-4.444	0.0002
γ_{O1O3}	-0.02896656	0.00681156	-4.253	0.0003
γ_{O1O4}	-0.00009831	0.00594010	-0.017	0.9869
γ_{O1A4}	-0.01118760	0.00444426	-2.517	0.0196
γ_{O1K1}	-0.00594938	0.00304735	-1.952	0.0637
γ_{O1E1}	-0.00902644	0.00320907	-2.813	0.0101
1/2 γ_{O2O2}	0.12207719	0.00798352	15.291	0.0001
γ_{O2O3}	-0.13385114	0.01695096	-7.896	0.0001
γ_{O2O4}	-0.04675512	0.00962793	-4.856	0.0001
γ_{O2A4}	-0.00719520	0.00874309	-0.823	0.4194
γ_{O2K1}	-0.02814958	0.00541762	-5.196	0.0001
γ_{O2E1}	-0.00630420	0.00457626	-1.378	0.1822
1/2 γ_{O3O3}	0.08896500	0.01215282	7.321	0.0001
γ_{O3O4}	0.01528570	0.01152300	1.327	0.1983
γ_{O3A4}	-0.01508504	0.01182632	-1.276	0.2154
γ_{O3K1}	-0.00057406	0.00670683	-0.086	0.9326
γ_{O3E1}	-0.01473888	0.00578132	-2.549	0.0183
1/2 γ_{O4O4}	0.01588087	0.00641976	2.474	0.0216
γ_{O4A4}	0.00911076	0.00796223	1.144	0.2648
γ_{O4K1}	-0.00955627	0.00522528	-1.829	0.0810

Table D.22: (continued)

coefficient	parameter estimate	standard error	t statistic	p-value
γ_{O4E1}	-0.00025149	0.00498022	0.050	0.9602
$1/2\ \gamma_{A4A4}$	0.01253957	0.00465424	2.694	0.0132
γ_{A4K1}	-0.00318203	0.00433194	-0.735	0.4704
γ_{A4E1}	0.00245998	0.00382668	0.643	0.5270
$1/2\ \gamma_{K1K1}$	0.03128576	0.00203968	15.339	0.0001
γ_{K1E1}	-0.01516020	0.00286993	-5.282	0.0001
$1/2\ \gamma_{E1E1}$	0.02125913	0.00195498	10.874	0.0001
ρ_{YO1}	-0.00404860	0.00360431	-1.123	0.2734
ρ_{YO2}	0.09336409	0.00601866	15.512	0.0001
ρ_{YO3}	-0.04897962	0.00664566	-7.370	0.0001
ρ_{YO4}	-0.00954670	0.00516046	-1.850	0.0778
ρ_{YA4}	-0.00506207	0.00411711	-1.230	0.2319
ρ_{YK1}	-0.03078920	0.00327734	-9.395	0.0001
ρ_{YE1}	0.00506210	0.00299566	1.690	0.1052

Table D.23: Summary Statistics for M.67

equation [a]	R^2	*MSE*	*df*
cost	0.9255	0.2553	31
O1	0.6976	0.0014	59
O2	0.8407	0.0011	59
O3	0.6789	0.0024	59
O4	0.4140	0.0021	59
A4	0.3697	0.0011	59
K1	0.6727	0.0008	59
system weighted	0.7487	1.2419	433

test for homotheticity	$F^{6}_{433} = 40.7802$	*p*-value = 0.0001
test for homogeneity	$F^{7}_{433} = 35.5841$	*p*-value = 0.0001

Note: a. statistics for each equation refer to first-stage estimation.

Table D.24: Estimated Shares - M.67

evaluated at	O1	O2	O3	O4	A4	K1	E1
average firm [a]	14.6758	37.7858	19.6223	8.2195	4.9502	8.1311	6.6153
	0.4620	0.4048	0.5957	0.5655	0.4013	0.3512	0.3956
point of approx.	15.4467	39.7328	16.2021	8.2770	4.7784	6.5118	9.0513
	0.5695	0.5542	0.7840	0.7239	0.5236	0.4452	0.4707

Note: a. the average firm is defined as the harmonic mean of the cost function variables.
b. standard errors of estimates are indicated in smaller type.

Table D.25: Price Elasticities Evaluated at the Average Shares - M.67

η_{kl}	O1	O2	O3	O4	A4	K1	E1
O1	-0.3277	0.2286	-0.0012	0.0815	-0.0267	0.0408	0.0047
	0.03459	0.03358	0.04641	0.04048	0.03028	0.02076	0.02187
O2	0.0888	0.0240	-0.1580	-0.0415	0.0305	0.0068	0.0495
	0.01304	0.04226	0.04486	0.02548	0.02314	0.01434	0.01211
O3	-0.0009	-0.3043	0.1030	0.1601	-0.0274	0.0784	-0.0090
	0.03471	0.08639	0.12387	0.05872	0.06027	0.03418	0.02946
O4	0.1456	-0.1910	0.3822	-0.5314	0.1604	-0.0350	0.0692
	0.07227	0.11714	0.14019	0.15621	0.09687	0.06357	0.06059
A4	-0.0792	0.2325	-0.1085	0.2662	-0.4439	0.0170	0.1159
	0.08978	0.17662	0.23890	0.16085	0.18804	0.08751	0.07730
K1	0.0736	0.0317	0.1892	-0.0353	0.0104	-0.1492	-0.1203
	0.03748	0.06663	0.08248	0.06426	0.05328	0.05017	0.03530
E1	0.0103	0.2826	-0.0266	0.0860	0.0867	-0.1479	-0.2911
	0.04851	0.06918	0.08739	0.07528	0.05785	0.04338	0.05910

Note: standard errors of estimates are indicated in smaller type.

Table D.26: Elasticities of Substitution Evaluated at the Average Shares

σ_{kl}	O1	O2	O3	O4	A4	K1	E1
O1	-2.2329	0.6051	-0.0059	0.9919	-0.5400	0.5014	0.0703
	0.23569	0.08887	0.23653	0.49243	0.61174	0.25537	0.33054
O2	0.6051	0.0636	-0.8053	-0.5054	0.6153	0.0838	0.7478
	0.08887	0.11183	0.22862	0.31000	0.46742	0.17633	0.18308
O3	-0.0059	-0.8053	0.5249	1.9477	-0.5530	0.9640	-0.1354
	0.23653	0.22862	0.63126	0.71445	1.21751	0.42036	0.44538
O4	0.9919	-0.5054	1.9478	-6.4650	3.2392	-0.4299	1.0463
	0.49243	0.31000	0.71445	1.90048	1.95689	0.78184	0.91591
A4	-0.5400	0.6153	-0.5530	3.2392	-8.9667	0.2095	1.7512
	0.61175	0.46742	1.21751	1.95689	3.79862	1.07623	1.16854
K1	0.5014	0.0838	0.9640	-0.4299	0.2095	-1.8344	-1.8184
	0.25537	0.17633	0.42036	0.78184	1.07623	0.61701	0.53354
E1	0.0703	0.7478	-0.1354	1.0463	1.7512	-1.8184	-4.4007
	0.33054	0.18308	0.44538	0.91591	1.16854	0.53354	0.89345

Note: standard errors of estimates are indicated in smaller type.

Table D.27: Price Elasticities at the Point of Approximation - M.67

η_{kl}	O1	O2	O3	O4	A4	K1	E1
O1	-0.3462 0.03286	0.2556 0.03190	-0.0255 0.04410	0.0821 0.03846	-0.0246 0.02877	0.0266 0.01973	0.0321 0.02078
O2	0.0994 0.01240	0.0118 0.04019	-0.1749 0.04266	-0.0349 0.02423	0.0297 0.02201	-0.0057 0.01364	0.0747 0.01152
O3	-0.0243 0.04204	-0.4288 0.10462	0.2602 0.15002	0.1771 0.07112	-0.0453 0.07299	0.0616 0.04139	-0.0005 0.03568
O4	0.1533 0.07177	-0.1676 0.11632	0.3467 0.13922	-0.5335 0.15512	0.1579 0.09620	-0.0503 0.06313	0.0936 0.06017
A4	-0.0797 0.09301	0.2468 0.18297	-0.1537 0.24750	0.2734 0.16663	-0.4274 0.19481	-0.0015 0.09066	0.1420 0.08008
K1	0.0631 0.04680	-0.0350 0.08320	0.1532 0.10300	-0.0640 0.08024	-0.0011 0.06653	0.0260 0.06265	-0.1423 0.04407
E1	0.0547 0.03545	0.3277 0.05056	-0.0008 0.06387	0.0856 0.05502	0.0750 0.04228	-0.1024 0.03171	-0.4397 0.04320

Note: standard errors of estimates are indicated in smaller type.

Table D.28: Elasticities of Substitution at the Point of Approximation

σ_{kl}	O1	O2	O3	O4	A4	K1	E1
O1	-2.2414 0.21276	0.6432 0.08030	-0.1574 0.27217	0.9923 0.46461	-0.5157 0.60212	0.4085 0.30296	0.3544 0.22953
O2	0.6432 0.08030	0.0297 0.10114	-1.0792 0.26331	-0.4217 0.29276	0.6210 0.46051	-0.0880 0.20939	0.8247 0.12725
O3	-0.1574 0.27217	-1.0792 0.26331	1.6060 0.92590	2.1398 0.85926	-0.9485 1.52757	0.9456 0.63569	-0.0050 0.39423
O4	0.9923 0.46461	-0.4217 0.29276	2.1398 0.85926	-6.4455 1.87416	3.3036 2.01319	-0.7730 0.96948	1.0336 0.66476
A4	-0.5157 0.60212	0.6210 0.46051	-0.9485 1.52757	3.3036 2.01319	-8.9438 4.07681	-0.0226 1.39221	1.5688 0.88477
K1	0.4085 0.30296	-0.0880 0.20939	0.9456 0.63569	-0.7730 0.96948	-0.0226 1.39221	0.3995 0.96204	-1.5721 0.48692
E1	0.3544 0.22953	0.8247 0.12725	-0.0050 0.39423	1.0336 0.66476	1.5688 0.88477	-1.5721 0.48692	-4.8583 0.47725

Note: standard errors of estimates are indicated in smaller type.

Table D.29: Parameter Estimates for Model M.148

coefficient	parameter estimate	standard error	t statistic	p-value
α_0	18.20900688	0.06411907	283.987	0.0001
α_Y	0.67773332	0.04841891	13.997	0.0001
δ_{YY}	0.01548882	0.01405696	1.102	0.2731
β_{O1}	0.22077760	0.00753401	29.304	0.0001
β_{O2}	0.16024689	0.00744884	21.513	0.0001
β_{O3}	0.26526140	0.01056061	25.118	0.0001
β_{O4}	0.10187462	0.00812906	12.532	0.0001
β_{A4}	0.05089262	0.00676029	7.528	0.0001
β_{K1}	0.08871425	0.00541783	16.375	0.0001
β_{E1}	0.11223262	0.00726244	15.454	0.0001
$1/2\ \gamma_{O1O1}$	0.05874647	0.00221401	26.534	0.0001
γ_{O1O2}	-0.00349643	0.00165527	-2.112	0.0371
γ_{O1O3}	-0.08288753	0.00529916	-15.642	0.0001
γ_{O1O4}	-0.00686716	0.00503159	-1.365	0.1753
γ_{O1A4}	-0.00516384	0.00410754	-1.257	0.2115
γ_{O1K1}	-0.00682816	0.00250670	-2.724	0.0076
γ_{O1E1}	-0.01224983	0.00260854	-4.696	0.0001
$1/2\ \gamma_{O2O2}$	0.01367066	0.00091424	14.953	0.0001
γ_{O2O3}	-0.00604518	0.00234730	-2.575	0.0114
γ_{O2O4}	-0.00600491	0.00185025	-3.245	0.0016
γ_{O2A4}	-0.00206593	0.00152758	-1.352	0.1792
γ_{O2K1}	-0.00405557	0.00112552	-3.603	0.0005
γ_{O2E1}	-0.00567330	0.00158353	-3.583	0.0005
$1/2\ \gamma_{O3O3}$	0.07916518	0.00603130	13.126	0.0001
γ_{O3O4}	-0.02633334	0.00909930	-2.894	0.0046
γ_{O3A4}	-0.00251668	0.00699938	-0.360	0.7199
γ_{O3K1}	-0.02382353	0.00410867	-5.798	0.0001
γ_{O3E1}	-0.01672411	0.00386797	-4.324	0.0001
$1/2\ \gamma_{O4O4}$	0.03190956	0.00535492	5.959	0.0001
γ_{O4A4}	-0.01343568	0.00649932	-2.067	0.0412
γ_{O4K1}	-0.00815008	0.00379516	-2.147	0.0341

Table D.29: (continued)

coefficient	parameter estimate	standard error	t statistic	p-value
γ_{O4E1}	-0.00302795	0.00314949	-0.961	0.3386
1/2 γ_{A4A4}	0.01331572	0.00374548	3.555	0.0006
γ_{A4K1}	-0.00670588	0.00322387	-2.080	0.0400
γ_{A4E1}	0.00325656	0.00263403	1.236	0.2191
1/2 γ_{K1K1}	0.03231005	0.00142087	22.740	0.0001
γ_{K1E1}	-0.01505689	0.00187414	-8.034	0.0001
1/2 γ_{E1E1}	0.02473777	0.00154518	16.010	0.0001
ρ_{YO1}	-0.00157370	0.00314021	-0.501	0.6173
ρ_{YO2}	0.01320452	0.00286557	4.608	0.0001
ρ_{YO3}	0.00978789	0.00460641	2.125	0.0360
ρ_{YO4}	0.00282365	0.00348092	0.811	0.4191
ρ_{YA4}	-0.00794562	0.00289882	-2.741	0.0072
ρ_{YK1}	-0.01922446	0.00242577	-7.925	0.0001
ρ_{YE1}	0.00292772	0.00300703	0.974	0.3325

Table D.30: Summary Statistics for M.148

equation [a]	R^2	*MSE*	*df*
cost	0.8639	0.2641	112
O1	0.6630	0.0024	140
O2	0.6177	0.0024	140
O3	0.6268	0.0047	140
O4	0.3152	0.0024	140
A4	0.1874	0.0016	140
K1	0.6074	0.0013	140
system weighted	0.7384	1.1407	1000
test for homotheticity	$F^{6}_{1000} = 14.1467$	p-value = 0.0001	
test for homogeneity	$F^{7}_{1000} = 12.1701$	p-value = 0.0001	

Note: a. statistics for each equation refer to first-stage estimation.

Table D.31: Estimated Shares - M.148

evaluated at	O1	O2	O3	O4	A4	K1	E1
average firm [a]	21.6697	9.0005	31.1014	11.3957	6.8275	10.2518	9.7534
	0.4018	0.3931	0.5608	0.3999	0.3300	0.2954	0.4281
point of approx.	22.0778	16.0247	26.5261	10.1875	5.0893	8.8714	11.2233
	0.7534	0.7449	1.0561	0.8129	0.6760	0.5418	0.7262

Note: a. the average firm is defined as the harmonic mean of the cost function variables.
b. standard errors of estimates are indicated in smaller type.

Table D.32: Price Elasticities Evaluated at the Average Shares - M.148

η_{kl}	O1	O2	O3	O4	A4	K1	E1
O1	-0.2411 0.02043	0.0739 0.00764	-0.0715 0.02445	0.0823 0.02322	0.0445 0.01896	0.0710 0.01157	0.0410 0.01204
O2	0.1779 0.01839	-0.6062 0.02032	0.2439 0.02608	0.0472 0.02056	0.0453 0.01697	0.0575 0.01251	0.0345 0.01759
O3	-0.0498 0.01704	0.0706 0.00755	-0.1799 0.03879	0.0293 0.02926	0.0602 0.02251	0.0259 0.01321	0.0438 0.01244
O4	0.1564 0.04415	0.0373 0.01624	0.0799 0.07985	-0.3260 0.09398	-0.0496 0.05703	0.0310 0.03330	0.0710 0.02764
A4	0.1411 0.06016	0.0598 0.02237	0.2742 0.10252	-0.0828 0.09519	-0.5417 0.10972	0.0043 0.04722	0.1452 0.03858
K1	0.1501 0.02445	0.0505 0.01098	0.0786 0.04008	0.0345 0.03702	0.0029 0.03145	-0.2672 0.02772	-0.0493 0.01828
E1	0.0911 0.02674	0.0318 0.01624	0.1395 0.03966	0.0829 0.03229	0.1017 0.02701	-0.0519 0.01922	-0.3952 0.03168

Note: standard errors of estimates are indicated in smaller type.

Table D.33: Elasticities of Substitution Evaluated at the Average Shares

σ_{kl}	O1	O2	O3	O4	A4	K1	E1
O1	-1.1126 0.09430	0.8207 0.08487	-0.2299 0.07863	0.7219 0.20376	0.6510 0.27763	0.6926 0.11284	0.4204 0.12342
O2	0.8207 0.08487	-6.7354 0.22572	0.7840 0.08385	0.4145 0.18040	0.6638 0.24859	0.5605 0.12198	0.3537 0.18039
O3	-0.2299 0.07863	0.7840 0.08385	-0.5785 0.12470	0.2570 0.25674	0.8815 0.32962	0.2528 0.12886	0.4487 0.12751
O4	0.7219 0.20376	0.4145 0.18040	0.2570 0.25674	-2.8609 0.82471	-0.7269 0.83535	0.3024 0.32486	0.7276 0.28336
A4	0.6510 0.27763	0.6638 0.24859	0.8815 0.32962	-0.7269 0.83535	-7.9336 1.60700	0.0419 0.46059	1.4890 0.39555
K1	0.6926 0.11284	0.5605 0.12198	0.2528 0.12886	0.3024 0.32486	0.0419 0.46059	-2.6059 0.27039	-0.5058 0.18743
E1	0.4204 0.12342	0.3537 0.18039	0.4487 0.12751	0.7276 0.28336	1.4890 0.39555	-0.5058 0.18743	-4.0519 0.32486

Note: standard errors of estimates are indicated in smaller type.

Table D.34: Price Elasticities at the Point of Approximation - M.148

η_{kl}	O1	O2	O3	O4	A4	K1	E1
O1	-0.2470	0.1444	-0.1102	0.0708	0.0275	0.0578	0.0568
	0.02006	0.00750	0.02400	0.02279	0.01861	0.01135	0.01182
O2	0.1990	-0.6691	0.2275	0.0644	0.0380	0.0634	0.0768
	0.01033	0.01141	0.01465	0.01155	0.00953	0.00702	0.00988
O3	-0.0917	0.1375	-0.1379	0.0026	0.0414	-0.0011	0.0492
	0.01998	0.00885	0.04547	0.03430	0.02639	0.01549	0.01458
O4	0.1534	0.1013	0.0068	-0.2717	-0.0810	0.0087	0.0825
	0.04939	0.01816	0.08932	0.10513	0.06380	0.03725	0.03092
A4	0.1193	0.1197	0.2158	-0.1621	-0.4258	-0.0431	0.1762
	0.08071	0.03002	0.13753	0.12771	0.14719	0.06335	0.05176
K1	0.1438	0.1145	-0.0033	0.0100	-0.0247	-0.1829	-0.0575
	0.02826	0.01269	0.04631	0.04278	0.03634	0.03203	0.02113
E1	0.1116	0.1097	0.1163	0.0749	0.0799	-0.0454	-0.4469
	0.02324	0.01411	0.03446	0.02806	0.02347	0.01670	0.02754

Note: standard errors of estimates are indicated in smaller type.

Table D.35: Elasticities of Substitution at the Point of Approximation

σ_{kl}	O1	O2	O3	O4	A4	K1	E1
O1	-1.1190	0.9012	-0.4153	0.6947	0.5404	0.6514	0.5056
	0.09085	0.04679	0.09049	0.22371	0.36557	0.12798	0.10527
O2	0.9012	-4.1756	0.8578	0.6322	0.7467	0.7147	0.6846
	0.04679	0.07121	0.05522	0.11334	0.18731	0.07917	0.08805
O3	-0.4153	0.8578	-0.5197	0.0255	0.8136	-0.0124	0.4382
	0.09049	0.05522	0.17143	0.33672	0.51848	0.17460	0.12992
O4	0.6947	0.6322	0.0255	-2.6668	-1.5914	0.0982	0.7352
	0.22371	0.11334	0.33672	1.03193	1.25357	0.41992	0.27546
A4	0.5404	0.7467	0.8136	-1.5914	-8.3670	-0.4853	1.5701
	0.36557	0.18731	0.51848	1.25357	2.89220	0.71405	0.46116
K1	0.6514	0.7147	-0.0124	0.0982	-0.4853	-2.0614	-0.5122
	0.12798	0.07917	0.17460	0.41992	0.71405	0.36108	0.18823
E1	0.5056	0.6846	0.4382	0.7352	1.5701	-0.5122	-3.9822
	0.10527	0.08805	0.12992	0.27546	0.46116	0.18823	0.24534

Note: standard errors of estimates are indicated in smaller type.

Appendix E

COST MODELS FOR GROUP G.877: PARAMETER ESTIMATES AND DERIVED ELASTICITIES

Table E.1: Parameter Estimates for Model M.853

coefficient	parameter estimate	standard error	*t* statistic	*p*-value
α_0	15.83615428	0.02189741	723.198	0.0001
α_Y	0.64753984	0.01724711	37.545	0.0001
δ_{YY}	0.05451874	0.00609031	8.952	0.0001
β_{O1}	0.25313668	0.00285732	88.592	0.0001
β_{O3}	0.30107865	0.00397643	75.716	0.0001
β_{O4}	0.12264931	0.00339280	36.150	0.0001
β_{A4}	0.09342473	0.00408121	22.891	0.0001
β_{K1}	0.14273044	0.00247925	57.570	0.0001
β_{E1}	0.08698019	0.00167305	51.989	0.0001
1/2 γ_{O1O1}	0.06662990	0.00119751	55.640	0.0001
γ_{O1O3}	-0.07046297	0.00310147	-22.719	0.0001
γ_{O1O4}	-0.02217620	0.00282467	-7.851	0.0001
γ_{O1A4}	-0.02115936	0.00324543	-6.520	0.0001
γ_{O1K1}	-0.01673988	0.00150472	-11.125	0.0001
γ_{O1E1}	-0.00272138	0.00116052	-2.345	0.0193
1/2 γ_{O3O3}	0.06659635	0.00351975	18.921	0.0001
γ_{O3O4}	-0.02337178	0.00526528	-4.439	0.0001

Table E.1: (continued)

coefficient	parameter estimate	standard error	t statistic	p-value
γ_{O3A4}	-0.01003236	0.00553216	-1.813	0.0701
γ_{O3K1}	-0.02138588	0.00245228	-8.721	0.0001
γ_{O3E1}	-0.00793972	0.00186580	-4.255	0.0001
1/2 γ_{O4O4}	0.04772105	0.00320683	14.881	0.0001
γ_{O4A4}	-0.02157003	0.00506051	-4.262	0.0001
γ_{O4K1}	-0.02101500	0.00237041	-8.866	0.0001
γ_{O4E1}	-0.00730909	0.00179223	-4.078	0.0001
1/2 γ_{A4A4}	0.04060152	0.00377766	10.748	0.0001
γ_{A4K1}	-0.02183723	0.00264919	-8.243	0.0001
γ_{A4E1}	-0.00660405	0.00201192	-3.282	0.0011
1/2 γ_{K1K1}	0.04737864	0.00090935	52.102	0.0001
γ_{K1E1}	-0.01377928	0.00100294	-13.739	0.0001
1/2 γ_{E1E1}	0.01917676	0.00052976	36.199	0.0001
ρ_{YO1}	0.01883101	0.00200131	9.409	0.0001
ρ_{YO3}	0.02306570	0.00284920	8.095	0.0001
ρ_{YO4}	-0.00417739	0.00238266	-1.753	0.0799
ρ_{YA4}	-0.01388802	0.00284519	-4.881	0.0001
ρ_{YK1}	-0.02818176	0.00175456	-16.062	0.0001
ρ_{YE1}	0.00435046	0.00114945	3.785	0.0002

Table E.2: Summary Statistics for M.853

equation [a]	R^2	*MSE*	*df*
cost	0.7733	0.2416	825
O1	0.6134	0.0042	846
O3	0.4375	0.0081	846
O4	0.2433	0.0057	846
A4	0.1822	0.0081	846
K1	0.6078	0.0032	846
system weighted	0.7348	1.0803	5090
test for homotheticity	$F^{5}_{5090} = 62.8589$	p-value = 0.0001	
test for homogeneity	$F^{6}_{5090} = 62.1286$	p-value = 0.0001	

Note: a. statistics for each equation refer to first-stage estimation.

Table E.3: Estimated Shares - M.853

evaluated at	O1	O2	O3	O4	A4	K1	E1
average firm [a]	21.6584	–	31.4406	13.9901	11.9998	14.1078	6.8033
	0.2208		0.3053	0.2569	0.3075	0.1924	0.1340
point of approx.	25.3137	–	30.1079	12.2649	9.3425	14.2730	8.6980
	0.2857		0.3976	0.3393	0.4081	0.2479	0.1673

Note: a. the average firm is defined as the harmonic mean of the cost function variables.
b. standard errors of estimates are indicated in smaller type.

Table E.4: Price Elasticities Evaluated at the Average Shares - M.853

η_{kl}	O1	O2	O3	O4	A4	K1	E1
O1	-0.1681 0.01106		-0.0109 0.01432	0.0375 0.01304	0.0223 0.01498	0.0638 0.00695	0.0555 0.00536
O2							
O3	-0.0075 0.00986		-0.2620 0.02239	0.0656 0.01675	0.0881 0.01760	0.0731 0.00780	0.0428 0.00593
O4	0.0581 0.02019		0.1474 0.03764	-0.1779 0.04584	-0.0342 0.03617	-0.0091 0.01694	0.0158 0.01281
A4	0.0403 0.02705		0.2308 0.04610	-0.0399 0.04217	-0.2033 0.06296	-0.0409 0.02208	0.0130 0.01677
K1	0.0979 0.01067		0.1628 0.01738	-0.0091 0.01680	-0.0348 0.01878	-0.1873 0.01289	-0.0296 0.00711
E1	0.1766 0.01706		0.1977 0.02743	0.0325 0.02634	0.0229 0.02957	-0.0615 0.01474	-0.3682 0.01557

Note: standard errors of estimates are indicated in smaller type.

Table E.5: Elasticities of Substitution Evaluated at the Average Shares

σ_{kl}	O1	O2	O3	O4	A4	K1	E1
O1	-0.7763 0.05106		-0.0348 0.04555	0.2681 0.09322	0.1859 0.12487	0.4521 0.04925	0.8153 0.07876
O2							
O3	-0.0348 0.04555		-0.8332 0.07121	0.4687 0.11970	0.7341 0.14663	0.5179 0.05529	0.6288 0.08723
O4	0.2681 0.09322		0.4687 0.11970	-1.2715 0.32769	-0.2849 0.30144	-0.0648 0.12010	0.2321 0.18830
A4	0.1859 0.12487		0.7341 0.14663	-0.2849 0.30144	-1.6942 0.52469	-0.2899 0.15649	0.1911 0.24644
K1	0.4521 0.04925		0.5179 0.05529	-0.0648 0.12010	-0.2899 0.15649	-1.3273 0.09138	-0.4356 0.10450
E1	0.8153 0.07876		0.6288 0.08723	0.2321 0.18830	0.1911 0.24644	-0.4356 0.10450	-5.4123 0.22891

Note: standard errors of estimates are indicated in smaller type.

Table E.6: Price Elasticities at the Point of Approximation - M.853

η_{kl}	O1	O2	O3	O4	A4	K1	E1
O1	-0.2204		0.0227	0.0350	0.0098	0.0766	0.0762
	0.00946		0.01225	0.01116	0.01282	0.00594	0.00458
O2							
O3	0.0191		-0.2565	0.0450	0.0601	0.0717	0.0606
	0.01030		0.02338	0.01749	0.01837	0.00815	0.00620
O4	0.0723		0.1105	-0.0992	-0.0824	-0.0286	0.0274
	0.02303		0.04293	0.05229	0.04126	0.01933	0.01461
A4	0.0267		0.1937	-0.1082	-0.0374	-0.0910	0.0163
	0.03474		0.05922	0.05417	0.08087	0.02836	0.02154
K1	0.1359		0.1512	-0.0246	-0.0596	-0.1934	-0.0096
	0.01054		0.01718	0.01661	0.01856	0.01274	0.00703
E1	0.2219		0.2098	0.0386	0.0175	-0.0157	-0.4721
	0.01334		0.02145	0.02061	0.02313	0.01153	0.01218

Note: standard errors of estimates are indicated in smaller type.

Table E.7: Elasticities of Substitution at the Point of Approximation

σ_{kl}	O1	O2	O3	O4	A4	K1	E1
O1	-0.8708		0.0755	0.2857	0.1053	0.5367	0.8764
	0.03738		0.04069	0.09098	0.13723	0.04165	0.05271
O2							
O3	0.0755		-0.8521	0.3671	0.6433	0.5023	0.6968
	0.04069		0.07766	0.14259	0.19668	0.05707	0.07125
O4	0.2857		0.3671	-0.8087	-0.8825	-0.2005	0.3149
	0.09098		0.14259	0.42636	0.44164	0.13541	0.16800
A4	0.1053		0.6433	-0.8825	-0.4003	-0.6376	0.1873
	0.13723		0.19668	0.44164	0.86562	0.19867	0.24759
K1	0.5367		0.5023	-0.2005	-0.6376	-1.3549	-0.1099
	0.04165		0.05707	0.13541	0.19867	0.08927	0.08079
E1	0.8764		0.6968	0.3149	0.1873	-0.1099	-5.4274
	0.05271		0.07125	0.16800	0.24759	0.08079	0.14005

Note: standard errors of estimates are indicated in smaller type.

Table E.8: Parameter Estimates for Model M.66

coefficient	parameter estimate	standard error	t statistic	p-value
α_0	15.48094557	0.05960091	259.743	0.0001
α_Y	0.58447274	0.04504350	12.976	0.0001
δ_{YY}	0.06943508	0.01348803	5.148	0.0001
β_{O1}	0.13656850	0.00873808	15.629	0.0001
β_{O3}	0.15200665	0.01102909	13.782	0.0001
β_{O4}	0.18080897	0.01736296	10.413	0.0001
β_{A4}	0.29772007	0.02200947	13.527	0.0001
β_{K1}	0.13368164	0.00946640	14.122	0.0001
β_{E1}	0.09921417	0.00737473	13.453	0.0001
1/2 γ_{O1O1}	0.03316258	0.00293534	11.298	0.0001
γ_{O1O3}	-0.06474205	0.00661474	-9.788	0.0001
γ_{O1O4}	0.00237848	0.00870170	0.273	0.7865
γ_{O1A4}	-0.00213545	0.01062268	-0.201	0.8420
γ_{O1K1}	-0.00386098	0.00430327	-0.897	0.3767
γ_{O1E1}	0.00203483	0.00341476	0.596	0.5557
1/2 γ_{O3O3}	0.02631082	0.00805348	3.267	0.0027
γ_{O3O4}	-0.00060911	0.01260846	-0.048	0.9618
γ_{O3A4}	0.01305356	0.01609869	0.811	0.4238
γ_{O3K1}	-0.00307789	0.00641690	-0.480	0.6350
γ_{O3E1}	0.00275383	0.00485831	0.567	0.5750
1/2 γ_{O4O4}	0.05263369	0.01189519	4.425	0.0001
γ_{O4A4}	-0.05419667	0.02164980	-2.503	0.0180
γ_{O4K1}	-0.03362369	0.00873209	-3.851	0.0006
γ_{O4E1}	-0.01921638	0.00684849	-2.806	0.0087
1/2 γ_{A4A4}	0.03105111	0.01650365	1.881	0.0696
γ_{A4K1}	-0.00359257	0.01049318	-0.342	0.7345
γ_{A4E1}	-0.01523111	0.00833397	-1.828	0.0776
1/2 γ_{K1K1}	0.02887391	0.00321656	8.977	0.0001
γ_{K1E1}	-0.01359271	0.00376604	-3.609	0.0011
1/2 γ_{E1E1}	0.02162577	0.00196487	11.006	0.0001

Table E.8: (continued)

coefficient	parameter estimate	standard error	t statistic	p-value
ρ_{YO1}	0.00462217	0.00415436	1.113	0.2747
ρ_{YO3}	0.00091977	0.00510148	0.180	0.8581
ρ_{YO4}	0.01340116	0.00826389	1.622	0.1153
ρ_{YA4}	-0.01903503	0.00955622	-1.992	0.0556
ρ_{YK1}	-0.01030729	0.00425135	-2.424	0.0216
ρ_{YE1}	0.01039922	0.00341795	3.043	0.0048

Table E.9: Summary Statistics for M.66

equation [a]	R^2	*MSE*	*df*
cost	0.9251	0.1123	38
O1	0.6952	0.0017	59
O3	0.5567	0.0023	59
O4	0.3403	0.0069	59
A4	0.2276	0.0090	59
K1	0.5066	0.0018	59
system weighted	0.7790	1.0519	368

test for homotheticity	$F^{5}_{368} = 4.9287$	p-value = 0.0003
test for homogeneity	$F^{6}_{368} = 8.3999$	p-value = 0.0001

Note: a. statistics for each equation refer to first-stage estimation.

Table E.10: Estimated Shares - M.66

evaluated at	O1	O2	O3	O4	A4	K1	E1
average firm [a]	12.5667	–	16.1079	19.2834	34.5122	10.3280	7.2018
	0.5050		0.5843	1.0130	1.1593	0.5177	0.4507
point of approx.	13.6569	–	15.2007	18.0809	29.7720	13.3682	9.9214
	0.8738		1.1029	1.7363	2.2009	0.9466	0.7375

Note: a. the average firm is defined as the harmonic mean of the cost function variables.
b. standard errors of estimates are indicated in smaller type.

Table E.11: Price Elasticities Evaluated at the Average Shares - M.66

η_{kl}	O1	O2	O3	O4	A4	K1	E1
O1	-0.3466		-0.3541	0.2118	0.3281	0.0726	0.0882
	0.04672		0.05264	0.06924	0.08453	0.03424	0.02717
O2							
O3	-0.2763		-0.5122	0.1891	0.4262	0.0842	0.0891
	0.04107		0.09999	0.07828	0.09994	0.03984	0.03016
O4	0.1380		0.1579	-0.2613	0.0641	-0.0711	-0.0276
	0.04513		0.06539	0.12337	0.11227	0.04529	0.03552
A4	0.1195		0.1989	0.0358	-0.4749	0.0929	0.0279
	0.03078		0.04665	0.06273	0.09564	0.03040	0.02415
K1	0.0883		0.1313	-0.1327	0.3103	-0.3376	-0.0596
	0.04167		0.06213	0.08455	0.10160	0.06229	0.03646
E1	0.1539		0.1993	-0.0740	0.1336	-0.0855	-0.3274
	0.04742		0.06746	0.09509	0.11571	0.05229	0.05457

Note: standard errors of estimates are indicated in smaller type.

Table E.12: Elasticities of Substitution Evaluated at the Average Shares

σ_{kl}	O1	O2	O3	O4	A4	K1	E1
O1	-2.7577		-2.1984	1.0982	0.9508	0.7025	1.2248
	0.37174		0.32678	0.35909	0.24493	0.33156	0.37731
O2							
O3	-2.1984		-3.1801	0.9804	1.2348	0.8150	1.2374
	0.32678		0.62078	0.40592	0.28959	0.38572	0.41880
O4	1.0982		0.9804	-1.3549	0.1856	-0.6883	-0.3837
	0.35909		0.40592	0.63979	0.32531	0.43845	0.49314
A4	0.9508		1.2348	0.1856	-1.3761	0.8992	0.3872
	0.24493		0.28959	0.32531	0.27712	0.29439	0.33530
K1	0.7025		0.8150	-0.6883	0.8992	-3.2686	-0.8275
	0.33156		0.38572	0.43845	0.29439	0.60310	0.50632
E1	1.2248		1.2374	-0.3837	0.3872	-0.8275	-4.5463
	0.37731		0.41880	0.49314	0.33530	0.50632	0.75767

Note: standard errors of estimates are indicated in smaller type.

Table E.13: Price Elasticities at the Point of Approximation - M.66

η_{kl}	O1	O2	O3	O4	A4	K1	E1
O1	-0.3778 0.04299		-0.3221 0.048435	0.1982 0.06372	0.2821 0.07778	0.1054 0.03151	0.1141 0.02500
O2							
O3	-0.2894 0.04352		-0.5018 0.10596	0.1768 0.08295	0.3836 0.10591	0.1134 0.04221	0.1173 0.03196
O4	0.1497 0.04813		0.1486 0.06973	-0.2370 0.13158	-0.0020 0.11974	-0.0523 0.04829	-0.0071 0.03788
A4	0.1294 0.03569		0.1959 0.05407	-0.0012 0.07272	-0.4937 0.11087	0.1216 0.03525	0.0481 0.02799
K1	0.1077 0.03219		0.1281 0.04800	-0.0707 0.06532	0.2709 0.07849	-0.4343 0.04812	-0.0025 0.02817
E1	0.1571 0.03442		0.1797 0.04897	-0.0129 0.06903	0.1442 0.08400	-0.0033 0.03796	-0.4648 0.03961

Note: standard errors of estimates are indicated in smaller type.

Table E.14: Elasticities of Substitution at the Point of Approximation

σ_{kl}	O1	O2	O3	O4	A4	K1	E1
O1	-2.7662 0.31477		-2.1187 0.31864	1.0963 0.35240	0.9475 0.26126	0.7885 0.23571	1.1502 0.25202
O2							
O3	-2.1187 0.31864		-3.3013 0.69709	0.9778 0.45875	1.2884 0.35573	0.8485 0.31579	1.1826 0.32214
O4	1.0963 0.35240		0.9778 0.45875	-1.3107 0.72772	-0.0068 0.40219	-0.3911 0.36127	-0.0712 0.38177
A4	0.9475 0.26126		1.2884 0.35573	-0.0068 0.40219	-1.6582 0.37239	0.9097 0.26365	0.4844 0.28214
K1	0.7885 0.23571		0.8485 0.31579	-0.3911 0.36127	0.9097 0.26365	-3.2490 0.35998	-0.0249 0.28395
E1	1.1502 0.25202		1.1826 0.32214	-0.0712 0.38177	0.4844 0.28214	-0.0249 0.28395	-4.6853 0.39922

Note: standard errors of estimates are indicated in smaller type.

Table E.15: Parameter Estimates for Model M.787

coefficient	parameter estimate	standard error	t statistic	p-value
α_0	15.85876546	0.02292780	691.683	0.0001
α_Y	0.65376398	0.01812755	36.065	0.0001
δ_{YY}	0.05505403	0.00669910	8.218	0.0001
β_{O1}	0.26120367	0.00261949	99.715	0.0001
β_{O3}	0.30866591	0.00370106	83.399	0.0001
β_{O4}	0.11905417	0.00341416	34.871	0.0001
β_{A4}	0.08446252	0.00312081	27.064	0.0001
β_{K1}	0.14060124	0.00257429	54.618	0.0001
β_{E1}	0.08601250	0.00172527	49.855	0.0001
1/2 γ_{O1O1}	0.06895523	0.00109858	62.768	0.0001
γ_{O1O3}	-0.07607282	0.00286268	-26.574	0.0001
γ_{O1O4}	-0.02402837	0.00281376	-8.540	0.0001
γ_{O1A4}	-0.01368691	0.00263866	-5.187	0.0001
γ_{O1K1}	-0.02040910	0.00149956	-13.610	0.0001
γ_{O1E1}	-0.00371326	0.00114473	-3.244	0.0012
1/2 γ_{O3O3}	0.06582310	0.00337634	19.495	0.0001
γ_{O3O4}	-0.02327037	0.00524067	-4.440	0.0001
γ_{O3A4}	0.00341469	0.00481839	0.709	0.4787
γ_{O3K1}	-0.02552244	0.00242927	-10.506	0.0001
γ_{O3E1}	-0.01019526	0.00183082	-5.569	0.0001
1/2 γ_{O4O4}	0.04668678	0.00329528	14.168	0.0001
γ_{O4A4}	-0.02032582	0.00466380	-4.358	0.0001
γ_{O4K1}	-0.01958999	0.00244131	-8.024	0.0001
γ_{O4E1}	-0.00615900	0.00183641	-3.354	0.0008
1/2 γ_{A4A4}	0.02534308	0.00305990	8.282	0.0001
γ_{A4K1}	-0.01640263	0.00233529	-7.024	0.0001
γ_{A4E1}	-0.00368549	0.00175342	-2.102	0.0359
1/2 γ_{K1K1}	0.04802192	0.00093938	51.121	0.0001
γ_{K1E1}	-0.01411968	0.00103433	-13.651	0.0001
1/2 γ_{E1E1}	0.01893635	0.00054788	34.563	0.0001

Table E.15: (continued)

coefficient	parameter estimate	standard error	t statistic	p-value
ρ_{YO1}	0.01781558	0.00188637	9.444	0.0001
ρ_{YO3}	0.02086984	0.00272799	7.650	0.0001
ρ_{YO4}	-0.00463189	0.00246293	-1.881	0.0604
ρ_{YA4}	-0.00722350	0.00225672	-3.201	0.0014
ρ_{YK1}	-0.03039623	0.00186973	-16.257	0.0001
ρ_{YE1}	0.00356619	0.00121756	2.929	0.0035

Table E.16: Summary Statistics for M.787

equation [a]	R^2	*MSE*	*df*
cost	0.7693	0.2461	759
O1	0.6861	0.0033	780
O3	0.4912	0.0066	780
O4	0.2322	0.0054	780
A4	0.1335	0.0044	780
K1	0.6129	0.0033	780
system weighted	0.7490	1.0818	4694
test for homotheticity	$F^{5}_{4694} = 59.1376$	*p*-value = 0.0001	
test for homogeneity	$F^{6}_{4694} = 57.2724$	*p*-value = 0.0001	

Note: a. statistics for each equation refer to first-stage estimation.

Table E.17: Estimated Shares - M.787

evaluated at	O1	O2	O3	O4	A4	K1	E1
average firm [a]	22.4922 0.2045	–	32.8218 0.2870	13.5241 0.2619	9.9742 0.2364	14.4190 0.2025	6.7687 0.1402
point of approx.	26.1204 0.2619	–	30.8666 0.3701	11.9054 0.3414	8.4463 0.3121	14.0601 0.2574	8.6013 0.1725

Note: a. the average firm is defined as the harmonic mean of the cost function variables.
b. standard errors of estimates are indicated in smaller type.

Table E.18: Price Elasticities Evaluated at the Average Shares - M.787

η_{kl}	O1	O2	O3	O4	A4	K1	E1
O1	-0.1619		-0.0100	0.0284	0.0389	0.0535	0.0512
	0.00977		0.01273	0.01251	0.01173	0.00667	0.00509
O2							
O3	-0.0069		-0.2707	0.0643	0.1102	0.0664	0.0366
	0.00872		0.02057	0.01597	0.01468	0.00740	0.00558
O4	0.0473		0.1562	-0.1743	-0.0506	-0.0007	0.0222
	0.02081		0.03875	0.04873	0.03449	0.01805	0.01358
A4	0.0877		0.3625	-0.0685	-0.3921	-0.0203	0.0307
	0.02645		0.04831	0.04676	0.06136	0.02341	0.01758
K1	0.0834		0.1512	-0.0006	-0.0140	-0.1897	-0.0302
	0.01040		0.01685	0.01693	0.01620	0.01303	0.00717
E1	0.1701		0.1776	0.0443	0.0453	-0.0644	-0.3728
	0.01691		0.02705	0.02713	0.02590	0.01528	0.01619

Note: standard errors of estimates are indicated in smaller type.

Table E.19: Elasticities of Substitution Evaluated at the Average Shares

σ_{kl}	O1	O2	O3	O4	A4	K1	E1
O1	-0.7199		-0.0305	0.2101	0.3899	0.3707	0.7561
	0.04343		0.03878	0.09250	0.11762	0.04624	0.07519
O2							
O3	-0.0305		-0.8247	0.4758	1.1043	0.4607	0.5411
	0.03878		0.06268	0.11806	0.14719	0.05133	0.08241
O4	0.2101		0.4758	-1.2891	-0.5068	-0.0046	0.3272
	0.09250		0.11806	0.36033	0.34574	0.12519	0.20061
A4	0.3899		1.1043	-0.5068	-3.9310	-0.1405	0.4541
	0.11762		0.14719	0.34574	0.61515	0.16238	0.25972
K1	0.3707		0.4607	-0.0046	-0.1405	-1.3157	-0.4467
	0.04624		0.05133	0.12519	0.16238	0.09037	0.10598
E1	0.7561		0.5411	0.3272	0.4541	-0.4467	-5.5075
	0.07519		0.08241	0.20061	0.25972	0.10598	0.23917

Note: standard errors of estimates are indicated in smaller type.

Table E.20: Price Elasticities at the Point of Approximation - M.787

η_{kl}	O1	O2	O3	O4	A4	K1	E1
O1	-0.2108 0.00841		0.0174 0.01096	0.0271 0.01077	0.0321 0.01010	0.0625 0.00574	0.0718 0.00438
O2							
O3	0.0148 0.00927		-0.2648 0.02188	0.0437 0.01698	0.0955 0.01561	0.0579 0.00787	0.0530 0.00593
O4	0.0594 0.02363		0.1132 0.04402	-0.0967 0.05536	-0.0863 0.03917	-0.0240 0.02051	0.0343 0.01543
A4	0.0992 0.03124		0.3491 0.05705	-0.1216 0.05522	-0.3154 0.07246	-0.0536 0.02765	0.0424 0.02076
K1	0.1161 0.01067		0.1271 0.01728	-0.0203 0.01736	-0.0322 0.01661	-0.1763 0.01336	-0.0144 0.00736
E1	0.2180 0.01331		0.1901 0.02129	0.0475 0.02135	0.0416 0.02039	-0.0236 0.01203	-0.4737 0.01274

Note: standard errors of estimates are indicated in smaller type.

Table E.21: Elasticities of Substitution at the Point of Approximation

σ_{kl}	O1	O2	O3	O4	A4	K1	E1
O1	-0.8071 0.03220		0.0565 0.03551	0.2273 0.09048	0.3796 0.11960	0.4443 0.04083	0.8347 0.05095
O2							
O3	0.0565 0.03551		-0.8580 0.07088	0.3668 0.14261	1.1310 0.18482	0.4119 0.05598	0.6160 0.06896
O4	0.2273 0.09048		0.3668 0.14261	-0.8118 0.46498	-1.0213 0.46380	-0.1703 0.14584	0.3985 0.17933
A4	0.3796 0.11960		1.1310 0.18482	-1.0213 0.46380	-3.7346 0.85785	-0.3812 0.19665	0.4927 0.24136
K1	0.4443 0.04083		0.4119 0.05598	-0.1703 0.14584	-0.3812 0.19665	-1.2539 0.09504	-0.1675 0.08553
E1	0.8347 0.05095		0.6160 0.06896	0.3985 0.17933	0.4927 0.24136	-0.1675 0.08553	-5.5070 0.14811

Note: standard errors of estimates are indicated in smaller type.

Table E.22: Parameter Estimates for Model M.51

coefficient	parameter estimate	standard error	t statistic	p-value
α_0	15.83317317	0.07042451	224.825	0.0001
α_Y	0.71717508	0.05565540	12.886	0.0001
δ_{YY}	0.04764456	0.01885385	2.527	0.0232
β_{O1}	0.20685548	0.01092442	18.935	0.0001
β_{O3}	0.23916897	0.01733649	13.796	0.0001
β_{O4}	0.12908628	0.01028560	12.550	0.0001
β_{A4}	0.12968616	0.01694174	7.655	0.0001
β_{K1}	0.18996771	0.01985135	9.570	0.0001
β_{E1}	0.10523539	0.00877899	11.987	0.0001
1/2 γ_{O1O1}	0.04481895	0.00260185	17.226	0.0001
γ_{O1O3}	-0.07738601	0.00707584	-10.937	0.0001
γ_{O1O4}	0.00388218	0.00442221	0.878	0.3939
γ_{O1A4}	-0.00393037	0.00722938	-0.544	0.5947
γ_{O1K1}	-0.01727999	0.00586198	-2.948	0.0100
γ_{O1E1}	0.00507628	0.00301408	1.684	0.1128
1/2 γ_{O3O3}	0.02584679	0.00997517	2.591	0.0205
γ_{O3O4}	0.02655589	0.01131600	2.347	0.0331
γ_{O3A4}	0.00410390	0.01506709	0.272	0.7890
γ_{O3K1}	-0.00220258	0.00993183	-0.222	0.8275
γ_{O3E1}	-0.00276480	0.00500779	-0.552	0.5890
1/2 γ_{O4O4}	0.00675807	0.00533972	1.266	0.2250
γ_{O4A4}	-0.00265497	0.00962851	-0.276	0.7865
γ_{O4K1}	-0.04020132	0.00610752	-6.582	0.0001
γ_{O4E1}	-0.00109791	0.00309909	-0.354	0.7281
1/2 γ_{A4A4}	0.02593600	0.00938088	2.765	0.0145
γ_{A4K1}	-0.04860451	0.01053358	-4.614	0.0003
γ_{A4E1}	-0.00078605	0.00515300	-0.153	0.8808
1/2 γ_{K1K1}	0.06491797	0.00583586	11.124	0.0001
γ_{K1E1}	-0.02154754	0.00455601	-4.729	0.0003
1/2 γ_{E1E1}	0.01056001	0.00151116	6.988	0.0001

Table E.22: (continued)

coefficient	parameter estimate	standard error	t statistic	p-value
ρ_{YO1}	0.01292323	0.00582598	2.218	0.0424
ρ_{YO3}	0.01453751	0.00892860	1.628	0.1243
ρ_{YO4}	0.00882876	0.00456335	1.935	0.0721
ρ_{YA4}	0.00203542	0.00802559	0.254	0.8032
ρ_{YK1}	-0.05021708	0.00948225	-5.296	0.0001
ρ_{YE1}	0.01189217	0.00454669	2.616	0.0195

Table E.23: Summary Statistics for M.51

equation ^{a}	R^2	*MSE*	*df*
cost	0.9616	0.1127	23
O1	0.7696	0.0018	44
O3	0.6951	0.0035	44
O4	0.5776	0.0008	44
A4	0.3961	0.0029	44
K1	0.4663	0.0046	44
system weighted	0.8256	1.2253	278

test for homotheticity $F^{5}_{278} = 7.0937$ p-value = 0.0001

test for homogeneity $F^{6}_{278} = 6.2353$ p-value = 0.0001

Note: a. statistics for each equation refer to first-stage estimation.

Table E.24: Estimated Shares - M.51

evaluated at	O1	O2	O3	O4	A4	K1	E1
average firm ^{a}	16.5236	–	22.9585	8.9181	8.6913	36.9053	6.0031
	0.5792		0.8186	0.4058	0.7471	0.9333	0.5014
point of approx.	20.6855	–	23.9169	12.9086	12.9686	18.9968	10.5235
	1.0924		1.7336	1.0286	1.6942	1.9851	0.8779

Note: a. the average firm is defined as the harmonic mean of the cost function variables.
b. standard errors of estimates are indicated in smaller type.

Table E.25: Price Elasticities Evaluated at the Average Shares - M.51

η_{kl}	O1	O2	O3	O4	A4	K1	E1
O1	-0.2923		-0.2388	0.1127	0.0631	0.2645	0.0908
	0.03149		0.04282	0.02676	0.04375	0.03548	0.01824
O2							
O3	-0.1718		-0.5453	0.2049	0.1048	0.3595	0.0480
	0.03082		0.08690	0.04929	0.06563	0.04326	0.02181
O4	0.2088		0.5274	-0.7593	0.0571	-0.0817	0.0477
	0.04959		0.12689	0.11975	0.10797	0.06848	0.03475
A4	0.1200		0.2768	0.0586	-0.3163	-0.1902	0.0510
	0.08318		0.17336	0.11078	0.21587	0.12120	0.05929
K1	0.1184		0.2236	-0.0198	-0.0448	-0.2791	0.0017
	0.01588		0.02691	0.01655	0.02854	0.03163	0.01235
E1	0.2498		0.1835	0.0709	0.0738	0.0101	-0.5882
	0.05021		0.08342	0.05163	0.08584	0.07589	0.05035

Note: standard errors of estimates are indicated in smaller type.

Table E.26: Elasticities of Substitution Evaluated at the Average Shares

σ_{kl}	O1	O2	O3	O4	A4	K1	E1
O1	-1.7689		-1.0399	1.2634	0.7263	0.7166	1.5118
	0.19059		0.18652	0.30010	0.50340	0.09613	0.30386
O2							
O3	-1.0399		-2.3749	2.2970	1.2057	0.9740	0.7994
	0.18652		0.37850	0.55268	0.75510	0.11722	0.36335
O4	1.2634		2.2970	-8.5137	0.6575	-0.2215	0.7949
	0.30010		0.55268	1.34277	1.24223	0.18557	0.57887
A4	0.7263		1.2057	0.6575	-3.6388	-0.5153	0.8493
	0.50340		0.75510	1.24223	2.48376	0.32840	0.98764
K1	0.7166		0.9740	-0.2215	-0.5153	-0.7564	0.0274
	0.09613		0.11722	0.18557	0.32840	0.08570	0.20565
E1	1.5118		0.7994	0.7949	0.8493	0.0274	-9.7974
	0.30386		0.36335	0.57887	0.98764	0.20565	0.83866

Note: standard errors of estimates are indicated in smaller type.

Table E.27: Price Elasticities at the Point of Approximation - M.51

η_{kl}	O1	O2	O3	O4	A4	K1	E1
O1	-0.3598		-0.1349	0.1479	0.1107	0.1064	0.1298
	0.02516		0.03421	0.02138	0.03495	0.02834	0.01457
O2							
O3	-0.1167		-0.5447	0.2401	0.1469	0.1808	0.0937
	0.02959		0.08342	0.04731	0.06300	0.04153	0.02094
O4	0.2369		0.4449	-0.7662	0.1091	-0.1215	0.0967
	0.03426		0.08766	0.08273	0.07459	0.04731	0.02401
A4	0.1766		0.2708	0.1086	-0.4703	-0.1848	0.0992
	0.05575		0.11618	0.07424	0.14467	0.08122	0.03973
K1	0.1159		0.2276	-0.0825	-0.1262	-0.1266	-0.0082
	0.03086		0.05228	0.03215	0.05545	0.06144	0.02398
E1	0.2551		0.2129	0.1187	0.1222	-0.0148	-0.6941
	0.02864		0.04759	0.02945	0.04897	0.04329	0.02872

Note: standard errors of estimates are indicated in smaller type.

Table E.28: Elasticities of Substitution at the Point of Approximation

σ_{kl}	O1	O2	O3	O4	A4	K1	E1
O1	-1.7394		-0.5642	1.1454	0.8535	0.5603	1.2332
	0.12161		0.14302	0.16561	0.26949	0.14918	0.13846
O2							
O3	-0.5642		-2.2774	1.8602	1.1323	0.9515	0.8902
	0.14302		0.34877	0.36653	0.48577	0.21860	0.19897
O4	1.1454		1.8602	-5.9356	0.8414	-0.6394	0.9192
	0.16561		0.36653	0.64090	0.57516	0.24906	0.22814
A4	0.8535		1.1323	0.8414	-3.6267	-0.9729	0.9424
	0.26949		0.48577	0.57516	1.11554	0.42757	0.37758
K1	0.5603		0.9515	-0.6394	-0.9729	-0.6663	-0.0778
	0.14918		0.21860	0.24906	0.42757	0.32343	0.22790
E1	1.2332		0.8902	0.9192	0.9424	-0.0778	-6.5954
	0.13846		0.19897	0.22814	0.37758	0.22790	0.27291

Note: standard errors of estimates are indicated in smaller type.

Table E.29: Parameter Estimates for Model M.736

coefficient	parameter estimate	standard error	*t* statistic	*p*-value
α_0	15.86269997	0.02328833	681.144	0.0001
α_Y	0.64537868	0.01825028	35.363	0.0001
δ_{YY}	0.05200859	0.00656267	7.925	0.0001
β_{O1}	0.26581844	0.00260141	102.183	0.0001
β_{O3}	0.31273867	0.00378846	82.550	0.0001
β_{O4}	0.12048782	0.00360546	33.418	0.0001
β_{A4}	0.08490991	0.00324952	26.130	0.0001
β_{K1}	0.13076543	0.00216217	60.479	0.0001
β_{E1}	0.08527973	0.00177301	48.099	0.0001
1/2 γ_{O1O1}	0.07102749	0.00113834	62.396	0.0001
γ_{O1O3}	-0.07709477	0.00300315	-25.671	0.0001
γ_{O1O4}	-0.02728990	0.00298141	-9.153	0.0001
γ_{O1A4}	-0.01517538	0.00277109	-5.476	0.0001
γ_{O1K1}	-0.01711879	0.00135956	-12.591	0.0001
γ_{O1E1}	-0.00537615	0.00119058	-4.516	0.0001
1/2 γ_{O3O3}	0.06856386	0.00349029	19.644	0.0001
γ_{O3O4}	-0.02718854	0.00548818	-4.954	0.0001
γ_{O3A4}	0.00185612	0.00502137	0.370	0.7118
γ_{O3K1}	-0.02508579	0.00235596	-10.648	0.0001
γ_{O3E1}	-0.00961474	0.00192580	-4.993	0.0001
1/2 γ_{O4O4}	0.04721860	0.00346292	13.635	0.0001
γ_{O4A4}	-0.02181299	0.00487544	-4.474	0.0001
γ_{O4K1}	-0.01107800	0.00234787	-4.718	0.0001
γ_{O4E1}	-0.00706777	0.00193193	-3.658	0.0003
1/2 γ_{A4A4}	0.02574827	0.00319317	8.064	0.0001
γ_{A4K1}	-0.01269808	0.00225555	-5.630	0.0001
γ_{A4E1}	-0.00366622	0.00183761	-1.995	0.0464
1/2 γ_{K1K1}	0.03989781	0.00078824	50.616	0.0001
γ_{K1E1}	-0.01381496	0.00094289	-14.652	0.0001
1/2 γ_{E1E1}	0.01976992	0.00057262	34.525	0.0001

Table E.29: (continued)

coefficient	parameter estimate	standard error	t statistic	p-value
ρ_{YO1}	0.01676617	0.00186382	8.996	0.0001
ρ_{YO3}	0.02025811	0.00277301	7.305	0.0001
ρ_{YO4}	-0.00686135	0.00257813	-2.661	0.0080
ρ_{YA4}	-0.00789785	0.00232879	-3.391	0.0007
ρ_{YK1}	-0.02557407	0.00156061	-16.387	0.0001
ρ_{YE1}	0.00330899	0.00124167	2.665	0.0079

Table E.30: Summary Statistics for M.736

equation [a]	R^2	*MSE*	*df*
cost	0.7700	0.2408	708
O1	0.7127	0.0031	729
O3	0.4830	0.0064	729
O4	0.2229	0.0057	729
A4	0.1332	0.0045	729
K1	0.5672	0.0021	729
system weighted	0.7589	1.0895	4388
test for homotheticity	$F^{5}_{4388} = 61.2145$	p-value = 0.0001	
test for homogeneity	$F^{6}_{4388} = 58.8902$	p-value = 0.0001	

Note: a. statistics for each equation refer to first-stage estimation.

Table E.31: Estimated Shares - M.736

evaluated at	O1	O2	O3	O4	A4	K1	E1
average firm [a]	22.9197	–	33.5106	13.8656	10.0576	12.8145	6.8319
	0.2028		0.2936	0.2766	0.2459	0.1691	0.1437
point of approx.	26.5818	–	31.2739	12.0488	8.4910	13.0765	8.5280
	0.2601		0.3788	0.3605	0.3250	0.2162	0.1773

Note: a. the average firm is defined as the harmonic mean of the cost function variables.
b. standard errors of estimates are indicated in smaller type.

Table E.32: Price Elasticities Evaluated at the Average Shares - M.736

η_{kl}	O1	O2	O3	O4	A4	K1	E1
O1	-0.1510		-0.0013	0.0196	0.0344	0.0535	0.0449
	0.00993		0.01310	0.01301	0.01209	0.00593	0.00519
O2							
O3	-0.0009		-0.2557	0.0575	0.1061	0.0533	0.0396
	0.00896		0.02083	0.01638	0.01498	0.00703	0.00575
O4	0.0324		0.1390	-0.1803	-0.0567	0.0483	0.0174
	0.02150		0.03958	0.04995	0.03516	0.01693	0.01393
A4	0.0783		0.3536	-0.0782	-0.3874	0.0019	0.0319
	0.02755		0.04993	0.04847	0.06350	0.02243	0.01827
K1	0.0956		0.1394	0.0522	0.0015	-0.2492	-0.0395
	0.01061		0.01839	0.01832	0.01760	0.01230	0.00736
E1	0.1505		0.1944	0.0352	0.0469	-0.0741	-0.3529
	0.01743		0.02819	0.02828	0.02690	0.01380	0.01676

Note: standard errors of estimates are indicated in smaller type.

Table E.33: Elasticities of Substitution Evaluated at the Average Shares

σ_{kl}	O1	O2	O3	O4	A4	K1	E1
O1	-0.6589		-0.0038	0.1413	0.3417	0.4171	0.6567
	0.04334		0.03910	0.09382	0.12021	0.04629	0.07603
O2							
O3	-0.0038		-0.7630	0.4149	1.0551	0.4158	0.5800
	0.03910		0.06216	0.11812	0.14899	0.05486	0.08412
O4	0.1413		0.4149	-1.3000	-0.5642	0.3765	0.2539
	0.09382		0.11812	0.36024	0.34961	0.13214	0.20394
A4	0.3417		1.0551	-0.5642	-3.8519	0.0148	0.4664
	0.12021		0.14899	0.34961	0.63134	0.17501	0.26743
K1	0.4171		0.4158	0.3765	0.0148	-1.9443	-0.5780
	0.04629		0.05486	0.13214	0.17501	0.09600	0.10770
E1	0.6567		0.5800	0.2539	0.4664	-0.5780	-5.1659
	0.07603		0.08412	0.20394	0.26743	0.10770	0.24537

Note: standard errors of estimates are indicated in smaller type.

Table E.34: Price Elasticities at the Point of Approximation - M.736

η_{kl}	O1	O2	O3	O4	A4	K1	E1
O1	-0.1998		0.0227	0.0178	0.0278	0.0664	0.0651
	0.00856		0.01130	0.01122	0.01042	0.00511	0.00448
O2							
O3	0.0193		-0.2488	0.0336	0.0908	0.0506	0.0545
	0.00960		0.02232	0.01755	0.01606	0.00753	0.00616
O4	0.0393		0.0871	-0.0957	-0.0961	0.0388	0.0266
	0.02474		0.04555	0.05748	0.04046	0.01949	0.01603
A4	0.0871		0.3346	-0.1364	-0.3086	-0.0188	0.0421
	0.03264		0.05914	0.05742	0.07521	0.02656	0.02164
K1	0.1349		0.1209	0.0358	-0.0122	-0.2590	-0.0204
	0.01040		0.01802	0.01795	0.01725	0.01206	0.00721
E1	0.2028		0.2000	0.0376	0.0419	-0.0312	-0.4511
	0.01396		0.02258	0.02265	0.02155	0.01106	0.01343

Note: standard errors of estimates are indicated in smaller type.

Table E.35: Elasticities of Substitution at the Point of Approximation

σ_{kl}	O1	O2	O3	O4	A4	K1	E1
O1	-0.7516		0.0726	0.1479	0.3276	0.5075	0.7628
	0.03222		0.03613	0.09309	0.12277	0.03911	0.05252
O2							
O3	0.0726		-0.7955	0.2785	1.0699	0.3866	0.6395
	0.03613		0.07137	0.14565	0.18910	0.05761	0.07221
O4	0.1479		0.2785	-0.7945	-1.1321	0.2969	0.3122
	0.09309		0.14565	0.47708	0.47655	0.14902	0.18802
A4	0.3277		1.0699	-1.1321	-3.6345	-0.1436	0.4937
	0.12277		0.18910	0.47655	0.88580	0.20314	0.25378
K1	0.5075		0.3866	0.2969	-0.1436	-1.9808	-0.2388
	0.03911		0.05761	0.14902	0.20314	0.09220	0.08455
E1	0.7628		0.6395	0.3122	0.4937	-0.2388	-5.2893
	0.05252		0.07221	0.18802	0.25378	0.08455	0.15747

Note: standard errors of estimates are indicated in smaller type.

Table E.36: Parameter Estimates for Model M.39

coefficient	parameter estimate	standard error	t statistic	p-value
α_0	15.97287776	0.11845525	134.843	0.0001
α_Y	0.60496918	0.13299339	4.549	0.0199
δ_{YY}	-0.10503800	0.09841092	-1.067	0.3641
β_{O1}	0.30781582	0.01741829	17.672	0.0004
β_{O3}	0.32509026	0.01045208	31.103	0.0001
β_{O4}	0.08590059	0.01333087	6.444	0.0076
β_{A4}	0.06617441	0.01324440	4.996	0.0154
β_{K1}	0.11598889	0.00890262	13.029	0.0010
β_{E1}	0.09903003	0.00900041	11.003	0.0016
1/2 γ_{O1O1}	0.07812375	0.00694826	11.244	0.0015
γ_{O1O3}	-0.10899862	0.00849750	-12.827	0.0010
γ_{O1O4}	-0.00098798	0.01070423	-0.092	0.9323
γ_{O1A4}	-0.01190103	0.01097398	-1.084	0.3575
γ_{O1K1}	-0.02152013	0.00618580	-3.479	0.0401
γ_{O1E1}	-0.01283973	0.00556503	-2.307	0.1043
1/2 γ_{O3O3}	0.06770723	0.00478823	14.140	0.0008
γ_{O3O4}	-0.00665291	0.00815248	-0.816	0.4742
γ_{O3A4}	-0.00555687	0.00787334	-0.706	0.5312
γ_{O3K1}	-0.00362430	0.00423576	-0.856	0.4551
γ_{O3E1}	-0.01058175	0.00363711	-2.909	0.0620
1/2 γ_{O4O4}	0.00972888	0.00720694	1.350	0.2699
γ_{O4A4}	-0.00669547	0.01100009	-0.609	0.5857
γ_{O4K1}	0.00080884	0.00564293	0.143	0.8951
γ_{O4E1}	-0.00593024	0.00460558	-1.288	0.2882
1/2 γ_{A4A4}	0.01901887	0.00740305	2.569	0.0826
γ_{A4K1}	-0.01178192	0.00570940	-2.064	0.1310
γ_{A4E1}	-0.00210246	0.00471873	-0.446	0.6861
1/2 γ_{K1K1}	0.02420781	0.00243992	9.922	0.0022
γ_{K1E1}	-0.01229812	0.00334149	-3.680	0.0347
1/2 γ_{E1E1}	0.02187616	0.00221551	9.874	0.0022

Table E.36: (continued)

coefficient	parameter estimate	standard error	t statistic	p-value
ρ_{YO1}	0.01907519	0.01114749	1.711	0.1856
ρ_{YO3}	0.01200726	0.00676019	1.776	0.1738
ρ_{YO4}	-0.00497892	0.00806759	-0.617	0.5808
ρ_{YA4}	-0.01923987	0.00838610	-2.294	0.1055
ρ_{YK1}	-0.01641558	0.00675370	-2.431	0.0933
ρ_{YE1}	0.00955192	0.00740419	1.290	0.2875

Table E.37: Summary Statistics for M.39

equation [a]	R^2	MSE	df
cost	0.8911	0.3352	11
O1	0.5442	0.0027	32
O3	0.8355	0.0010	32
O4	0.2231	0.0014	32
A4	0.3713	0.0014	32
K1	0.5932	0.0010	32
system weighted	0.8070	1.1398	206

test for homotheticity $F^{5}_{206} = 2.6581$ p-value = 0.0236

test for homogeneity $F^{6}_{206} = 2.4126$ p-value = 0.0283

Note: a. statistics for each equation refer to first-stage estimation.

Table E.38: Estimated Shares - M.39

evaluated at	O1	O2	O3	O4	A4	K1	E1
average firm [a]	46.0498	–	21.9490	8.5488	6.5330	8.6994	8.2199
	0.8280		0.5017	0.5945	0.6033	0.5103	0.5825
point of approx.	30.7816	–	32.5090	8.5901	6.6174	11.5989	9.9030
	1.7418		1.0452	1.3331	1.3244	0.8903	0.9000

Note: a. the average firm is defined as the harmonic mean of the cost function variables.
b. standard errors of estimates are indicated in smaller type.

Table E.39: Price Elasticities Evaluated at the Average Shares - M.39

η_{kl}	O1	O2	O3	O4	A4	K1	E1
O1	-0.2002		-0.0172	0.0833	0.0395	0.0403	0.0543
	0.03018		0.01845	0.02325	0.02383	0.01343	0.01208
O2							
O3	-0.0361		-0.1636	0.0552	0.0400	0.0705	0.0340
	0.03872		0.04363	0.03714	0.03587	0.01930	0.01657
O4	0.4489		0.1417	-0.6869	-0.0130	0.0965	0.0128
	0.12521		0.09536	0.16861	0.12867	0.06601	0.05387
A4	0.2783		0.1344	-0.0170	-0.3524	-0.0934	0.0500
	0.16798		0.12052	0.16838	0.22664	0.08739	0.07223
K1	0.2131		0.1778	0.0948	-0.0701	-0.3565	-0.0592
	0.07111		0.04869	0.06487	0.06563	0.05609	0.03841
E1	0.3043		0.0908	0.0133	0.0398	-0.0626	-0.3855
	0.06770		0.04425	0.05603	0.05741	0.04065	0.05391

Note: standard errors of estimates are indicated in smaller type.

Table E.40: Elasticities of Substitution Evaluated at the Average Shares

σ_{kl}	O1	O2	O3	O4	A4	K1	E1
O1	-0.4348		-0.0784	0.9749	0.6044	0.4628	0.6608
	0.06553		0.08407	0.27191	0.36477	0.15441	0.14702
O2							
O3	-0.0784		-0.7452	0.6454	0.6125	0.8102	0.4135
	0.08407		0.19878	0.43448	0.54907	0.22183	0.20159
O4	0.9749		0.6454	-8.0351	-0.1988	1.1088	0.1561
	0.27191		0.43448	1.97227	1.96959	0.75877	0.65541
A4	0.6044		0.6125	-0.1988	-5.3946	-1.0731	0.6085
	0.36477		0.54907	1.96959	3.46906	1.00458	0.87871
K1	0.4628		0.8102	1.1088	-1.0731	-4.0976	-0.7198
	0.15441		0.22183	0.75877	1.00458	0.64480	0.46729
E1	0.6608		0.4135	0.1561	0.6085	-0.7198	-4.6902
	0.14702		0.20159	0.65541	0.87871	0.46729	0.65580

Note: standard errors of estimates are indicated in smaller type.

Table E.41: Price Elasticities at the Point of Approximation - M.39

η_{kl}	O1	O2	O3	O4	A4	K1	E1
O1	-0.1846 0.04515		-0.0290 0.02761	0.0827 0.03478	0.0275 0.03565	0.0461 0.02010	0.0573 0.01808
O2							
O3	-0.0275 0.02614		-0.2584 0.02946	0.0654 0.02508	0.0491 0.02422	0.1048 0.01303	0.0665 0.01119
O4	0.2963 0.12461		0.2476 0.09491	-0.6876 0.16780	-0.0118 0.12806	0.1254 0.06569	0.0300 0.05362
A4	0.1280 0.16583		0.2411 0.11898	-0.0153 0.16623	-0.3590 0.22374	-0.0621 0.08628	0.0673 0.07131
K1	0.1223 0.05333		0.2938 0.03652	0.0929 0.04865	-0.0354 0.04922	-0.4666 0.04207	-0.0070 0.02881
E1	0.1782 0.05620		0.2182 0.03673	0.0260 0.04651	0.0449 0.04765	-0.0082 0.03374	-0.4592 0.04474

Note: standard errors of estimates are indicated in smaller type.

Table E.42: Elasticities of Substitution at the Point of Approximation

σ_{kl}	O1	O2	O3	O4	A4	K1	E1
O1	-0.5997 0.14666		-0.0893 0.08492	0.9626 0.40483	0.4157 0.53874	0.3972 0.17326	0.5788 0.18256
O2							
O3	-0.0893 0.08492		-0.7948 0.09061	0.7618 0.29194	0.7417 0.36599	0.9039 0.11233	0.6713 0.11298
O4	0.9626 0.40483		0.7618 0.29194	-8.0044 1.95339	-0.1779 1.93513	1.0812 0.56636	0.3029 0.54140
A4	0.4157 0.53875		0.7417 0.36599	-0.1779 1.93513	-5.4253 3.38112	-0.5350 0.74385	0.6792 0.72006
K1	0.3973 0.17326		0.9039 0.11233	1.0812 0.56636	-0.5350 0.74385	-4.0228 0.36272	-0.0707 0.29091
E1	0.5788 0.18256		0.6713 0.11298	0.3029 0.54140	0.6792 0.72006	-0.0707 0.29091	-4.6366 0.45183

Note: standard errors of estimates are indicated in smaller type.

Table E.43: Parameter Estimates for Model M.697

coefficient	parameter estimate	standard error	t statistic	p-value
α_0	15.85916429	0.02323397	682.585	0.0001
α_Y	0.64251446	0.01786565	35.964	0.0001
δ_{YY}	0.05221248	0.00630659	8.279	0.0001
β_{O1}	0.25925538	0.00258023	100.477	0.0001
β_{O3}	0.31588088	0.00405256	77.946	0.0001
β_{O4}	0.12059738	0.00384686	31.350	0.0001
β_{A4}	0.08665294	0.00348636	24.855	0.0001
β_{K1}	0.13296580	0.00224045	59.348	0.0001
β_{E1}	0.08464762	0.00183848	46.042	0.0001
1/2 γ_{O1O1}	0.06586868	0.00115522	57.018	0.0001
γ_{O1O3}	-0.07033023	0.00321266	-21.892	0.0001
γ_{O1O4}	-0.02703662	0.00316579	-8.540	0.0001
γ_{O1A4}	-0.01318750	0.00295171	-4.468	0.0001
γ_{O1K1}	-0.01542083	0.00139550	-11.050	0.0001
γ_{O1E1}	-0.00576217	0.00120978	-4.763	0.0001
1/2 γ_{O3O3}	0.06771787	0.00357740	18.929	0.0001
γ_{O3O4}	-0.03098419	0.00578898	-5.352	0.0001
γ_{O3A4}	0.00248018	0.00526857	0.471	0.6380
γ_{O3K1}	-0.02694420	0.00243607	-11.061	0.0001
γ_{O3E1}	-0.00965730	0.00201462	-4.794	0.0001
1/2 γ_{O4O4}	0.04990451	0.00370577	13.467	0.0001
γ_{O4A4}	-0.02250246	0.00516292	-4.358	0.0001
γ_{O4K1}	-0.01239037	0.00245780	-5.041	0.0001
γ_{O4E1}	-0.00689536	0.00203311	-3.392	0.0007
1/2 γ_{A4A4}	0.02526559	0.00340128	7.428	0.0001
γ_{A4K1}	-0.01420131	0.00235635	-6.027	0.0001
γ_{A4E1}	-0.00312008	0.00193387	-1.613	0.1071
1/2 γ_{K1K1}	0.04125768	0.00081108	50.867	0.0001
γ_{K1E1}	-0.01355864	0.00096560	-14.042	0.0001
1/2 γ_{E1E1}	0.01949677	0.00058927	33.086	0.0001

Table E.43: (continued)

coefficient	parameter estimate	standard error	t statistic	p-value
ρ_{YO1}	0.01548857	0.00180461	8.583	0.0001
ρ_{YO3}	0.02110459	0.00290032	7.277	0.0001
ρ_{YO4}	-0.00681967	0.00267820	-2.546	0.0111
ρ_{YA4}	-0.00677353	0.00242737	-2.790	0.0054
ρ_{YK1}	-0.02601134	0.00158473	-16.414	0.0001
ρ_{YE1}	0.00301138	0.00126223	2.386	0.0173

Table E.44: Summary Statistics for M.697

equation [a]	R^2	*MSE*	*df*
cost	0.7838	0.2237	669
O1	0.6546	0.0027	690
O3	0.4415	0.0067	690
O4	0.2201	0.0058	690
A4	0.1264	0.0046	690
K1	0.5786	0.0021	690
system weighted	0.7569	1.0987	4154
test for homotheticity	$F^{5}_{4154} = 58.9748$	p-value = 0.0001	
test for homogeneity	$F^{6}_{4154} = 57.6066$	p-value = 0.0001	

Note: a. statistics for each equation refer to first-stage estimation.

Table E.45: Estimated Shares - M.697

evaluated at	O1	O2	O3	O4	A4	K1	E1
average firm [a]	21.6334	–	34.1453	14.1646	10.2561	13.0458	6.7549
	0.1958		0.3084	0.2880	0.2566	0.1726	0.1476
point of approx.	25.9255	–	31.5881	12.0597	8.6653	13.2966	8.4648
	0.2580		0.4053	0.3847	0.3486	0.2240	0.1839

Note: a. the average firm is defined as the harmonic mean of the cost function variables.
b. standard errors of estimates are indicated in smaller type.

Table E.46: Price Elasticities Evaluated at the Average Shares - M.697

η_{kl}	O1	O2	O3	O4	A4	K1	E1
O1	-0.1747		0.0164	0.0167	0.0416	0.0592	0.0409
	0.01068		0.01485	0.01463	0.01364	0.00645	0.00559
O2							
O3	0.0104		-0.2619	0.0509	0.1098	0.0516	0.0393
	0.00941		0.02095	0.01695	0.01543	0.00713	0.00590
O4	0.0255		0.1227	-0.1537	-0.0563	0.0430	0.0189
	0.02235		0.04087	0.05232	0.03645	0.01735	0.01435
A4	0.0878		0.3656	-0.0778	-0.4047	-0.0080	0.0371
	0.02878		0.05137	0.05034	0.06633	0.02298	0.01886
K1	0.0981		0.1349	0.0467	-0.0063	-0.2370	-0.0364
	0.01070		0.01867	0.01884	0.01806	0.01243	0.00740
E1	0.1310		0.1985	0.0396	0.0564	-0.0703	-0.3552
	0.01791		0.02982	0.03010	0.02863	0.01429	0.01745

Note: standard errors of estimates are indicated in smaller type.

Table E.47: Elasticities of Substitution Evaluated at the Average Shares

σ_{kl}	O1	O2	O3	O4	A4	K1	E1
O1	-0.8076		0.0479	0.1177	0.4056	0.4536	0.6057
	0.04937		0.04349	0.10331	0.13304	0.04945	0.08279
O2							
O3	0.0479		-0.7670	0.3594	1.0708	0.3951	0.5813
	0.04349		0.06137	0.11969	0.15045	0.05469	0.08735
O4	0.1177		0.3594	-1.0852	-0.5490	0.3295	0.2793
	0.10331		0.11969	0.36940	0.35539	0.13301	0.21249
A4	0.4056		1.0708	-0.5490	-3.9464	-0.0614	0.5496
	0.13304		0.15045	0.35539	0.64671	0.17611	0.27915
K1	0.4536		0.3951	0.3295	-0.0614	-1.8170	-0.5386
	0.04945		0.05469	0.13301	0.17611	0.09531	0.10957
E1	0.6057		0.5813	0.2793	0.5496	-0.5386	-5.2582
	0.08279		0.08735	0.21249	0.27915	0.10957	0.25829

Note: standard errors of estimates are indicated in smaller type.

Table E.48: Price Elasticities at the Point of Approximation - M.697

η_{kl}	O1	O2	O3	O4	A4	K1	E1
O1	-0.2326 0.00891		0.0446 0.01239	0.0163 0.01221	0.0358 0.01139	0.0735 0.00538	0.0624 0.00467
O2							
O3	0.0366 0.01017		-0.2554 0.02265	0.0225 0.01833	0.0945 0.01668	0.0477 0.00771	0.0541 0.00638
O4	0.0351 0.02625		0.0590 0.04800	-0.0518 0.06146	-0.0999 0.04281	0.0302 0.02038	0.0275 0.01686
A4	0.1071 0.03406		0.3445 0.06080	-0.1391 0.05958	-0.3302 0.07850	-0.0309 0.02719	0.0486 0.02232
K1	0.1433 0.01050		0.1132 0.01832	0.0274 0.01848	-0.0202 0.01772	-0.2465 0.01220	-0.0173 0.00726
E1	0.1912 0.01429		0.2018 0.02380	0.0391 0.02402	0.0498 0.02285	-0.0272 0.01141	-0.4547 0.01392

Note: standard errors of estimates are indicated in smaller type.

Table E.49: Elasticities of Substitution at the Point of Approximation

σ_{kl}	O1	O2	O3	O4	A4	K1	E1
O1	-0.8972 0.03438		0.1412 0.03923	0.1353 0.10126	0.4130 0.13139	0.5527 0.04048	0.7374 0.05513
O2							
O3	0.1412 0.03923		-0.8084 0.07171	0.1866 0.15196	1.0906 0.19248	0.3585 0.05800	0.6388 0.07535
O4	0.1353 0.10126		0.1867 0.15196	-0.4294 0.50960	-1.1533 0.49405	0.2273 0.15327	0.3245 0.19916
A4	0.4130 0.13139		1.0906 0.19248	-1.1533 0.49405	-3.8106 0.90595	-0.2326 0.20451	0.5746 0.26365
K1	0.5527 0.04048		0.3585 0.05800	0.2273 0.15327	-0.2326 0.20451	-1.8535 0.09175	-0.2047 0.08579
E1	0.7374 0.05513		0.6388 0.07535	0.3245 0.19916	0.5746 0.26365	-0.2047 0.08579	-5.3716 0.16448

Note: standard errors of estimates are indicated in smaller type.

Table E.50: Parameter Estimates for Model M.44

coefficient	parameter estimate	standard error	t statistic	p-value
α_0	16.15390660	0.08425749	191.721	0.0001
α_Y	0.55730978	0.07164972	7.778	0.0001
δ_{YY}	-0.00224889	0.03021428	-0.074	0.9425
β_{O1}	0.24925494	0.00927185	26.883	0.0001
β_{O3}	0.49995073	0.01306332	38.271	0.0001
β_{O4}	0.07433726	0.01147141	6.480	0.0002
β_{A4}	0.03245819	0.01314766	2.469	0.0388
β_{K1}	0.09616072	0.00728390	13.202	0.0001
β_{E1}	0.04783816	0.00588358	8.131	0.0001
1/2 γ_{O1O1}	0.04075377	0.00292635	13.927	0.0001
γ_{O1O3}	-0.04919117	0.00742023	-6.629	0.0002
γ_{O1O4}	-0.00409571	0.00674789	-0.607	0.5607
γ_{O1A4}	-0.01876964	0.00758291	-2.475	0.0384
γ_{O1K1}	-0.00338728	0.00351289	-0.964	0.3632
γ_{O1E1}	-0.00606374	0.00293527	-2.066	0.0727
1/2 γ_{O3O3}	0.04157577	0.01135341	3.662	0.0064
γ_{O3O4}	-0.03129667	0.01532596	-2.042	0.0754
γ_{O3A4}	0.01548629	0.01402264	1.104	0.3015
γ_{O3K1}	-0.00651773	0.00574199	-1.135	0.2892
γ_{O3E1}	-0.01163227	0.00466959	-2.491	0.0375
1/2 γ_{O4O4}	0.02428785	0.00783818	3.099	0.0147
γ_{O4A4}	0.00062122	0.01109386	0.056	0.9567
γ_{O4K1}	-0.01138980	0.00502225	-2.268	0.0531
γ_{O4E1}	-0.00241473	0.00429667	-0.562	0.5895
1/2 γ_{A4A4}	0.00532752	0.00798396	0.667	0.5234
γ_{A4K1}	-0.01488958	0.00555234	-2.682	0.0279
γ_{A4E1}	0.00689667	0.00481920	1.431	0.1903
1/2 γ_{K1K1}	0.02241612	0.00225401	9.945	0.0001
γ_{K1E1}	-0.00864785	0.00250978	-3.446	0.0088
1/2 γ_{E1E1}	0.01093096	0.00141071	7.749	0.0001

Table E.50: (continued)

coefficient	parameter estimate	standard error	t statistic	p-value
ρ_{YO1}	0.00258739	0.00379173	0.682	0.5143
ρ_{YO3}	0.00478716	0.00590499	0.811	0.4410
ρ_{YO4}	-0.00568734	0.00527841	-1.077	0.3127
ρ_{YA4}	0.00743222	0.00577237	1.288	0.2339
ρ_{YK1}	-0.00813410	0.00434820	-1.871	0.0983
ρ_{YE1}	-0.00098534	0.00297498	-0.331	0.7490

Table E.51: Summary Statistics for M.44

equation [a]	R^2	*MSE*	*df*
cost	0.9100	0.2127	16
O1	0.8567	0.0007	37
O3	0.5089	0.0012	37
O4	0.3644	0.0012	37
A4	0.2591	0.0015	37
K1	0.6241	0.0010	37
system weighted	0.7064	1.1266	236

test for homotheticity	$F^{5}_{236} = 1.0100$	p-value = 0.4130
test for homogeneity	$F^{6}_{236} = 0.8420$	p-value = 0.5386

Note: a. statistics for each equation refer to first-stage estimation.

Table E.52: Estimated Shares - M.44

evaluated at	O1	O2	O3	O4	A4	K1	E1
average firm [a]	15.5571	–	57.9820	8.1322	5.2778	9.0355	4.0154
	0.3941		0.5315	0.5120	0.5804	0.4665	0.3659
point of approx.	24.9255	–	49.9951	7.4337	3.2458	9.6161	4.7838
	0.9272		1.3063	1.1471	1.3148	0.7284	0.5884

Note: a. the average firm is defined as the harmonic mean of the cost function variables.
b. standard errors of estimates are indicated in smaller type.

Table E.53: Price Elasticities Evaluated at the Average Shares - M.44

η_{kl}	O1	O2	O3	O4	A4	K1	E1
O1	-0.3205		0.2636	0.0550	-0.0679	0.0686	0.0012
	0.03762		0.04770	0.04338	0.04874	0.02258	0.01887
O2							
O3	0.0707		-0.2768	0.0274	0.0795	0.0791	0.0201
	0.01280		0.03916	0.02643	0.02418	0.00990	0.00805
O4	0.1052		0.1950	-0.3214	0.0604	-0.0497	0.0105
	0.08298		0.18846	0.19277	0.13642	0.06176	0.05284
A4	-0.2001		0.8732	0.0931	-0.7453	-0.1918	0.1708
	0.14368		0.26569	0.21020	0.30255	0.10520	0.09131
K1	0.1181		0.5077	-0.0447	-0.1120	-0.4135	-0.0556
	0.03888		0.06355	0.05558	0.06145	0.04989	0.02778
E1	0.0046		0.2901	0.0212	0.2245	-0.1250	-0.4154
	0.07310		0.11629	0.10701	0.12002	0.06250	0.07026

Note: standard errors of estimates are indicated in smaller type.

Table E.54: Elasticities of Substitution Evaluated at the Average Shares

σ_{kl}	O1	O2	O3	O4	A4	K1	E1
O1	-2.0602		0.4547	0.6763	-1.2860	0.7590	0.0290
	0.24182		0.08226	0.53337	0.92353	0.24991	0.46988
O2							
O3	0.4547		-0.4773	0.3363	1.5060	0.8756	0.5000
	0.08226		0.06754	0.32503	0.45823	0.10960	0.20057
O4	0.6763		0.3363	-3.9516	1.1450	-0.5501	0.2610
	0.53337		0.32503	2.37042	2.58474	0.68350	1.31581
A4	-1.2860		1.5061	1.1447	-14.1220	-2.1223	4.2540
	0.92353		0.45823	2.58474	5.73241	1.16432	2.27400
K1	0.7590		0.8756	-0.5501	-2.1220	-4.5760	-1.3840
	0.24991		0.10960	0.68350	1.16432	0.55219	0.69176
E1	0.0293		0.5004	0.2605	4.2540	-1.3836	-10.3450
	0.46989		0.20057	1.31581	2.27400	0.69176	1.74988

Note: standard errors of estimates are indicated in smaller type.

Table E.55: Price Elasticities at the Point of Approximation - M.44

η_{kl}	O1	O2	O3	O4	A4	K1	E1
O1	-0.4237		0.3026	0.0579	-0.0428	0.0826	0.0235
	0.02348		0.02977	0.02707	0.03042	0.01409	0.01178
O2							
O3	0.1509		-0.3337	0.0117	0.0634	0.0831	0.0246
	0.01484		0.04542	0.03066	0.02805	0.01149	0.00934
O4	0.1942		0.0789	-0.2722	0.0408	-0.0571	0.0154
	0.09077		0.20617	0.21088	0.14924	0.06756	0.05780
A4	-0.3290		0.9771	0.0935	-0.6393	-0.3626	0.2603
	0.23362		0.43202	0.34179	0.49195	0.17106	0.14847
K1	0.2140		0.4322	-0.0441	-0.1224	-0.4376	-0.0421
	0.03653		0.05971	0.05223	0.05774	0.04688	0.02610
E1	0.1225		0.2568	0.0239	0.1766	-0.0846	-0.4952
	0.06136		0.09761	0.08982	0.10074	0.05246	0.05898

Note: standard errors of estimates are indicated in smaller type.

Table E.56: Elasticities of Substitution at the Point of Approximation

σ_{kl}	O1	O2	O3	O4	A4	K1	E1
O1	-1.7000		0.6053	0.7790	-1.3200	0.8587	0.4910
	0.09420		0.05955	0.36418	0.93730	0.14656	0.24617
O2							
O3	0.6053		-0.6675	0.1579	1.9540	0.8644	0.5140
	0.05955		0.09085	0.41238	0.86410	0.11944	0.19524
O4	0.7790		0.1579	-3.6619	1.2570	-0.5934	0.3210
	0.36418		0.41238	2.83682	4.59780	0.70258	1.20823
A4	-1.3200		1.9543	1.2575	-19.6950	-3.7705	5.4420
	0.93728		0.86413	4.59782	15.15650	1.77891	3.10367
K1	0.8587		0.8644	-0.5934	-3.7700	-4.5509	-0.8800
	0.14656		0.11944	0.70258	1.77890	0.48752	0.54559
E1	0.4915		0.5136	0.3210	5.4420	-0.8799	-10.3510
	0.24617		0.19524	1.20823	3.10370	0.54559	1.23287

Note: standard errors of estimates are indicated in smaller type.

Table E.57: Parameter Estimates for Model M.653

coefficient	parameter estimate	standard error	t statistic	p-value
α_0	15.83280519	0.02382461	664.557	0.0001
α_Y	0.64905786	0.01818579	35.690	0.0001
δ_{YY}	0.05902392	0.00637346	9.261	0.0001
β_{O1}	0.25617081	0.00264811	96.737	0.0001
β_{O3}	0.30755679	0.00385077	79.869	0.0001
β_{O4}	0.12491401	0.00396942	31.469	0.0001
β_{A4}	0.08947953	0.00353728	25.296	0.0001
β_{K1}	0.13549335	0.00226288	59.877	0.0001
β_{E1}	0.08638551	0.00190876	45.257	0.0001
1/2 γ_{O1O1}	0.06786457	0.00123570	54.920	0.0001
γ_{O1O3}	-0.06251519	0.00325235	-19.222	0.0001
γ_{O1O4}	-0.03159712	0.00334442	-9.448	0.0001
γ_{O1A4}	-0.01591909	0.00310596	-5.125	0.0001
γ_{O1K1}	-0.01867170	0.00145741	-12.812	0.0001
γ_{O1E1}	-0.00702604	0.00127656	-5.504	0.0001
1/2 γ_{O3O3}	0.06216249	0.00327615	18.974	0.0001
γ_{O3O4}	-0.02826528	0.00557306	-5.072	0.0001
γ_{O3A4}	0.00132785	0.00496375	0.268	0.7892
γ_{O3K1}	-0.02550035	0.00233380	-10.927	0.0001
γ_{O3E1}	-0.00937201	0.00195760	-4.787	0.0001
1/2 γ_{O4O4}	0.05168349	0.00383941	13.461	0.0001
γ_{O4A4}	-0.02381628	0.00534252	-4.458	0.0001
γ_{O4K1}	-0.01315534	0.00253186	-5.196	0.0001
γ_{O4E1}	-0.00653297	0.00213042	-3.067	0.0023
1/2 γ_{A4A4}	0.02852468	0.00347112	8.218	0.0001
γ_{A4K1}	-0.01519489	0.00241939	-6.280	0.0001
γ_{A4E1}	-0.00344694	0.00200315	-1.721	0.0858
1/2 γ_{K1K1}	0.04315169	0.00083626	51.601	0.0001
γ_{K1E1}	-0.01378111	0.00099574	-13.840	0.0001
1/2 γ_{E1E1}	0.02007954	0.00062142	32.313	0.0001

Table E.57: (continued)

coefficient	parameter estimate	standard error	t statistic	p-value
ρ_{YO1}	0.01754732	0.00186419	9.413	0.0001
ρ_{YO3}	0.01928462	0.00276090	6.985	0.0001
ρ_{YO4}	-0.00567757	0.00277242	-2.048	0.0410
ρ_{YA4}	-0.00710486	0.00247339	-2.873	0.0042
ρ_{YK1}	-0.02706244	0.00160356	-16.876	0.0001
ρ_{YE1}	0.00301292	0.00131381	2.293	0.0222

Table E.58: Summary Statistics for M.653

equation [a]	R^2	*MSE*	*df*
cost	0.7905	0.2187	625
O1	0.6551	0.0027	646
O3	0.3383	0.0057	646
O4	0.2271	0.0059	646
A4	0.1557	0.0045	646
K1	0.6054	0.0020	646
system weighted	0.7630	1.0861	3890
test for homotheticity	$F^{5}_{3890} = 63.8453$	p-value = 0.0001	
test for homogeneity	$F^{6}_{3890} = 63.7011$	p-value = 0.0001	

Note: a. statistics for each equation refer to first-stage estimation.

Table E.59: Estimated Shares - M.653

evaluated at	O1	C2	O3	O4	A4	K1	E1
average firm [a]	22.0613		32.6070	14.5381	10.5665	13.2947	6.9323
	0.2019		0.2940	0.2992	0.2625	0.1744	0.1533
point of approx.	25.6171		30.7557	12.4914	8.9480	13.5493	8.6386
	0.2648		0.3851	0.3969	0.3537	0.2263	0.1909

Note: a. the average firm is defined as the harmonic mean of the cost function variables.
b. standard errors of estimates are indicated in smaller type.

Table E.60: Price Elasticities Evaluated at the Average Shares - M.653

η_{kl}	O1	O2	O3	O4	A4	K1	E1
O1	-0.1642 0.01120		0.0427 0.01474	0.0022 0.01516	0.0335 0.01408	0.0483 0.00661	0.0375 0.00579
O2							
O3	0.0289 0.00997		-0.2927 0.02009	0.0587 0.01709	0.1097 0.01522	0.0547 0.00716	0.0406 0.00600
O4	0.0033 0.02300		0.1317 0.03833	-0.1436 0.05282	-0.0582 0.03675	0.0425 0.01742	0.0244 0.01465
A4	0.0700 0.02939		0.3386 0.04698	-0.0800 0.05056	-0.3544 0.06570	-0.0109 0.02290	0.0367 0.01896
K1	0.0802 0.01096		0.1343 0.01755	0.0464 0.01904	-0.0086 0.01820	-0.2179 0.01258	-0.0343 0.00749
E1	0.1193 0.01841		0.1909 0.02824	0.0511 0.03073	0.0559 0.02890	-0.0659 0.01436	-0.3514 0.01793

Note: standard errors of estimates are indicated in smaller type.

Table E.61: Elasticities of Substitution Evaluated at the Average Shares

σ_{kl}	O1	O2	O3	O4	A4	K1	E1
O1	-0.7441 0.05078		0.1310 0.04521	0.0148 0.10428	0.3171 0.13324	0.3634 0.04969	0.5406 0.08347
O2							
O3	0.1310 0.04521		-0.8975 0.06163	0.4037 0.11756	1.0385 0.14407	0.4118 0.05384	0.5854 0.08660
O4	0.0148 0.10428		0.4037 0.11756	-0.9878 0.36331	-0.5504 0.34778	0.3194 0.13099	0.3518 0.21139
A4	0.3171 0.13324		1.0385 0.14407	-0.5504 0.34778	-3.3543 0.62178	-0.0816 0.17222	0.5294 0.27347
K1	0.3634 0.04969		0.4118 0.05384	0.3194 0.13099	-0.0816 0.17222	-1.6390 0.09463	-0.4953 0.10804
E1	0.5406 0.08347		0.5854 0.08660	0.3518 0.21139	0.5294 0.27347	-0.4953 0.10804	-5.0686 0.25862

Note: standard errors of estimates are indicated in smaller type.

Table E.62: Price Elasticities at the Point of Approximation - M.653

η_{kl}	O1	O2	O3	O4	A4	K1	E1
O1	-0.2140 0.00965		0.0635 0.01270	0.0016 0.01306	0.0273 0.01212	0.0626 0.00569	0.0590 0.00498
O2							
O3	0.0529 0.01057		-0.2882 0.02130	0.0330 0.01812	0.0938 0.01614	0.0526 0.00759	0.0559 0.00637
O4	0.0032 0.02677		0.0813 0.04462	-0.0476 0.06147	-0.1012 0.04277	0.0302 0.02027	0.0341 0.01706
A4	0.0783 0.03471		0.3224 0.05547	-0.1413 0.05971	-0.2730 0.07758	-0.0343 0.02704	0.0479 0.02239
K1	0.1184 0.01076		0.1194 0.01722	0.0278 0.01869	-0.0227 0.01786	-0.2276 0.01234	-0.0153 0.00735
E1	0.1748 0.01478		0.1991 0.02266	0.0493 0.02466	0.0496 0.02319	-0.0240 0.01153	-0.4487 0.01439

Note: standard errors of estimates are indicated in smaller type.

Table E.63: Elasticities of Substitution at the Point of Approximation

σ_{kl}	O1	O2	O3	O4	A4	K1	E1
O1	-0.8353 0.03766		0.2065 0.04128	0.0126 0.10452	0.3055 0.13550	0.4621 0.04199	0.6825 0.05769
O2							
O3	0.2065 0.04128		-0.9371 0.06927	0.2643 0.14506	1.0483 0.18037	0.3881 0.05600	0.6473 0.07368
O4	0.0126 0.10452		0.2643 0.14506	-0.3809 0.49212	-1.1308 0.47798	0.2227 0.14959	0.3946 0.19743
A4	0.3055 0.13550		1.0483 0.18037	-1.1308 0.47798	-3.0504 0.86707	-0.2533 0.19956	0.5541 0.25915
K1	0.4621 0.04199		0.3881 0.05600	0.2227 0.14959	-0.2533 0.19956	-1.6794 0.09110	-0.1774 0.08507
E1	0.6825 0.05769		0.6473 0.07368	0.3946 0.19743	0.5541 0.25915	-0.1774 0.08507	-5.1945 0.16655

Note: standard errors of estimates are indicated in smaller type.

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